Unit and Measurements

EXERCISES

ELEMENTARY

Q.1 (2)
Velocity depends on length and time, so cannot be taken as base quantities.

Q.2 (3)
Light year is a unit of distance, which is cover by light in a year.

Q.3 (2)
System is NOT based on unit of mass, length and time alone,
This system is based on all 7 Fundamental physical quantities and 2 supplymentary physical quantities.

Q.4 (3)
Unit of pressure in S.I. System is pascal.

Q.5 (4)

$$[\text{magnetisation}] = \frac{[\text{magnetic field}]}{[\text{Length}]}$$

Q.6 (4)
Unit of K.E. $\Rightarrow$ Kg $m^2/S^2 \Rightarrow$ Kg m/s^2.m
$\Rightarrow \text{Newton} \times \text{m}$
Unit of viscosity is poise.
(3) (ML^2T^{-2})
(4) (Joule/m^2) = (N/m)

Q.7 (3)

$$\text{Force} \times \frac{\text{Displacement}}{\text{Time}}$$

$$= M^1L^1T^{-2} \times \frac{L^1}{T^1} = M^1L^2T^{-3}$$

So, Dimension of length here is 2.

Q.8 (2)
P = mvm $\rightarrow$ mass
v $\rightarrow$ velocity
Dimesion of [P] = $[MLT^{-1}]$

Q.9 (2)
Boltz mann's const. (k) $\rightarrow$ J $\rightarrow$ Joule
K $\rightarrow$ Kelvin
Unit $\rightarrow$ J/k

$$\text{Dimension} = \frac{M^1L^2T^{-2}}{K^1} = M^1L^2T^{-2}K^{-1}$$

Q.10 (2)
Find out Dimension of each physical quantity in all option.
AM = $Mvr^2 = ML^2T^{-1}$
Dimension of Torque $(\tau) = \vec{r} \times \vec{F}$ $\rightarrow$ Force = MLT^{-2}
$(\tau) = L^1 \times M^1L^1T^{-2}$
It is also a dimension of Energy. = ML^2T^{-2}
Power = W/t = $ML^2T^{-2}/T = ML^2T^{-3}$

Q.11 (2)
Dimension of Pressure = $M^1L^{-1}T^{-2}$
= Force/Area
It is same as energy per unit volume

$$= \frac{\text{Energy}}{\text{Volume}} = \frac{M^1L^2T^{-2}}{L^3} = M^1L^{-1}T^{-2}$$

Q.12 (3)
Because same physical quantities are added and subtracted.
$[at] = M^0L^0T^0$
$[a]\,T = M^0L^0T^0$, $[a] = M^0L^0T^{-1}$

Q.13 (3)
All the terms in the equation must have the dimension of force
$\therefore$ [A sin C t] = MLT^{-2}
$\Rightarrow [A]\,[M^0L^0T^0] = MLT^{-2}$
$\Rightarrow [A] = MLT^{-2}$
Similarly, $[B] = MLT^{-2}$

$$\therefore \frac{[A]}{[B]} = M^oL^oT^o$$

Again $[Ct] = M^oL^oT^o \quad \Rightarrow [C] = T^{-1}$
$[Dx] = MLT^o \Rightarrow [D] = L^{-1}$

$$\Rightarrow \frac{[C]}{[D]} = M^oL^1T^{-1}.$$

Q.14 (2)
It is obvious

Q.15 (1)

$[g] = LT^{-2}$ and numerical value $\propto \frac{1}{\text{unit}}$

Q.16 (4)
We know n, u, = n_2u_2
when $n_1 > n_2$ when $n_1 < n_2$ or then $u_1 < u_2$ then $u_1 > u_2$

So, we can say $n \propto \frac{1}{u}$

Q.17 (4)
$n_1u_1 = n_2u_2$

$1m^2 = n(xm)^2$

$n = \frac{1}{x^2}$

Q.18 (1)

$$n_2 = 13600\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^{-3}$$

$$= 13600\left[\frac{1000}{1}\right]^1\left[\frac{100}{1}\right]^{-3}$$

n_2 = 13.6 gcm^{-3}

Q.19 (2) $\therefore E = \frac{1}{2}mv^2$

$\therefore$ % Error in K.E.
= % error in mass + 2 × % error in velocity
= 2 + 2 × 3 = 8 %

Q.20 (2)

Q.21 (2)
Number of significant figures are 3, because 10^3 is decimal multiplier.

Q.22 (2)

$\because V = \frac{4}{3}\pi r^3$

$\therefore$ % error is volume $= 3 \times \%$ error in radius
$= 3 \times 1$ = 3%

Q.23 (3)
Mean time period T = 2.00 *sec*
& Mean absolute error = ΔT= 0.05 *sec*.
To express maximum estimate of error, the time period should be written as (2.00 ± 0.05) *sec*

Q.24 (3) % error in velocity = %error in L + %error in t

$$= \frac{0.2}{13.8} \times 100 + \frac{0.3}{4} \times 100$$

= 1.44 + 7.5 = 8.94 %

Q.25 (3)

Q.26 (1)

$\frac{1}{20} = 0.05$

$\therefore$ Decimal equivalent upto 3 significant figures is 0.0500

Q.27 (2)

Q.28 (1)

Since percentage increase in length = 2 %

Hence, percentage increase in area of square sheet

$= 2 \times 2\%$ = 4%

JEE MAIN
OBJECTIVE QUESTIONS

Q.1 (1)

Q.2 (1)
Kilogram is not a physical quantity, its a unit.

Q.3 (3)
PARSEC is a unit of distance.
It is used in astronmiccal science.

Q.4 (3)
S.I. unit of energy is Joule.

Q.5 (2)
SI unit of universal gravitational constant G is -

We know $F = \frac{GM_1M_2}{R^2}$

Here M_1 and M_2 are mass
R = Distance between them M_1 and M_2
F = Force

$$G = \frac{FR^2}{M_1M_2} = \frac{N - m^2}{kg^2}$$

So, Unit of G = N–m^2 kg^{-2}

Q.6 (2)
Surface Tension (T) :-

$T = \frac{J}{A} = \frac{J}{m^2}$

So S.I. unit of surface tension is joule/m^{+2}

Q.7 (1)

$$F = \eta\left(\frac{dv}{dx}\right)A$$

$$\frac{kgm}{sec^2} = \eta.\frac{m/sec}{m} \times m^2$$

$$\Rightarrow \eta = kg\,m^{-1}S^{-1}$$

Q.8 (4)

Here ρ is specific resistance.

$$R = \frac{\rho l}{A} \Rightarrow ohm = \frac{\rho m}{m^2} \qquad \Rightarrow \rho = ohm \times m$$

Q.9 (4)

Q.10 (1)

Here i = current

A = crossectional Area

M = iA

= Amp. m^2

Q.11 (4)

Unit of universal gas constant (R)

PV = nRT P $\rightarrow$ Pressure

V $\rightarrow$ Volume

$$R = \frac{PV}{nT}$$ T $\rightarrow$ Temperature

$$= \frac{N/m^2 \times m^3}{mol. \times K}$$ R $\rightarrow$ Univ. Gas. Const.

n $\rightarrow$ No. of male

$$= \frac{N - m}{mol. \times K} = Joule\,K^{-1}mol^{-1}$$

{n – m = joule}

Q.12 (4)

Stefan-Constant(σ)

Unit $\rightarrow$ $w/m^2\text{-}k^4 = wm^{-2}k^{-4}$

Q.13 (3)

S.I. unit of the angular acceleration is rad/s^2.

α = angular velocity/time

Q.14 (3)

Angular Frequency (f) $= \frac{1}{T} = M^oL^oT^{-1}$

So, here dimension in length is zero

Q.15 (2)

AM = mvr

$[AM] = [MLT^{-1}L] = [ML^2T^{-1}]$

Q.16 (2)

magnetic flux density = $\frac{weber}{metre^2} = \frac{ML^2T^{-2}A^{-1}}{L^2}$

Q.17 (4)

Find dimension in all options.

Here stress = Force/Area

$$= \frac{M^1L^1T^{-2}}{L^2}$$

stress = $[M^1L^{-1}T^{-2}]$

Q.18 (4)

$$\frac{VT}{I \times \frac{V}{I} \times \frac{Q}{V} \times V} = \frac{T}{Q} = \frac{T}{AT} = A^{-1}$$

Q.19 (3)

$$\int \frac{dx}{\sqrt{2ax - x^2}} = a^n \sin^{-1}\left[\frac{x}{a} - 1\right].$$

L.H.S. is the dimensionless as

denominator $2ax - x^2$ must have the dimension of $[x]^2$ ($\because$ we can add or substract only if quantities have same dimension)

$$\therefore \left[\sqrt{2ax - x^2}\right] = [x]$$

Also, dx has the dimension of [x]

$\therefore \frac{x\,dx}{\sqrt{2ax - x^2}}$ is having dimension L

Equating the dimension of L.H.S. & R.H.S. we have $[a^n] = M^0L^1T^0$

{ $\therefore \sin^{-1}\left(\frac{x}{a} - 1\right)$ must be dimensionless

$\therefore$ n = 1

Q.20 (1)

$$[\alpha] = \left[\frac{2ma}{\beta}\right] \text{ and } \left[\frac{2\beta\ell}{ma}\right] = 1$$

$$[\alpha] = \left[\frac{ma}{\beta}\right] \qquad \left[\frac{ma}{\beta}\right] = \ell$$

$[\alpha] = L$

Q.21 (2)

$[v] = [k]\,[\lambda^a \rho^b g^c] \Rightarrow LT^{-1} = L^a M^b L^{-3b} L^c T^{-2c}$

$\Rightarrow LT^{-1} = M^b L^{a-3b+c} T^{-2c}$

$\Rightarrow$ a = ½, b = 0, c = ½

so, $v^2 = kg\lambda$

Q.22 (4)

By checking each option.

$$\frac{V^2}{rg} = \frac{\left[L^1T^{-1}\right]^2}{[L^1][L^1T^{-2}]}$$

$$= \frac{L^2T^{-2}}{L^2T^{-2}} = [M^oL^oT^o]$$

Q.23 (1)

$G = 6.67 \times 10^{-11}$ N m^2 (kg)$^{-2}$

$= 6.67 \times 10^{-11} \times 10^5$ dyne $\times 100^2$ cm^2 / $(10^3)^2$ g^2 = 6.67×10^{-8} dyne-cm^2-g^{-2}

Q.24 (3)

$$P = \frac{W}{t}$$

Watt = Joule/sec.

Joule = Watt-sec.

One watt-hour = 1 watt×60×60 sec

1 Hour=60×60sec. = 3600 watt-sec

= 3600 Joule

= 3.6×10^3 Joule

Q.25 (1)

Given

P = 10^6 dyne/cm^2

$n_1u_1 = n_2u_2$

$$n_1\left[M_1^1L_1^{-1}T_1^{-2}\right] = 10^6\left[M_2^1L_2^{-1}T_2^{-2}\right]$$

$$n_1 = 10^6\left[\frac{M_2}{M_1}\right]^1\left[\frac{L_2}{L_1}\right]^{-1}\left[\frac{T_2}{T_1}\right]^{-2}$$

$$= 10^6\left[\frac{1}{1000}\right]^1\left[\frac{1}{100}\right]^{-1}$$

$$\Rightarrow 10^6 \times \frac{10^2}{10^3} = 10^5\ N/m^2$$

Q.26 (2)

ρ = 2g/cm^3

$n_1u_1 = n_2u_2$

$$n_1\left[M_1^1L_1^{-3}\right] = 2\left[M_2^1L_2^{-3}\right]$$

$$n_1 = 2\left[\frac{M_2}{M_1}\right]^1\left[\frac{L_2}{L_1}\right]^{-3} = 2\left[\frac{10^{-3}}{1}\right]^1\left[\frac{10^{-2}}{1}\right]^{-3}$$

= $2\times10^{-3}\times10^6$

= 2×10^3 Kg/m^3

Q.27 (1)

A = ℓb = 10.0 × 1.00 = 10.00

$$\frac{\Delta A}{A} = \frac{\Delta \ell}{\ell} + \frac{\Delta b}{b}$$

$$\frac{\Delta A}{10.00} = \frac{0.1}{10.0} + \frac{0.01}{1.00}$$

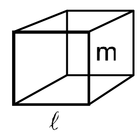
$$\Rightarrow \Delta A = 10.00\left(\frac{1}{100}+\frac{1}{100}\right) = 10.00\left(\frac{2}{100}\right)$$

= ± 0.2 cm^2.

Q.28 (4)

$$\left(\frac{\Delta A}{A}\right)_{min} = \left|\left(\frac{\Delta\ell}{\ell} - \frac{\Delta b}{b}\right)\right| = \left|\frac{1}{100} - \frac{1}{100}\right| = 0$$

Q.29 (2)



$$\rho = \frac{m}{V} = \frac{m}{\ell^3}$$

Given : $\frac{\Delta m}{m} = \pm$ 2% $= \pm 2 \times 10^{-2}$ $\frac{\Delta\ell}{\ell} = \pm 1\%$

$= \pm 1 \times 10^{-2}$

$$\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + 3\frac{\Delta\ell}{\ell}$$

$= 2 \times 10^{-2} + 3 \times 10^{-2} = 5 \times 10^{-2}$ = 5%

Q.30 (1)

$$g = 4\pi^2\frac{\ell}{T^2}$$

$\frac{\Delta\ell}{\ell}$ = 2% $= \pm 2 \times 10^{-2}$

$\frac{\Delta T}{T} = \pm 3\% = \pm 3 \times 10^{-2}$

$$\Rightarrow \frac{\Delta g}{g} = \frac{\Delta\ell}{\ell} + \frac{2\Delta T}{T}$$

$= 2 \times 10^{-2} + 2 \times 3 \times 10^{-2} = 8 \times 10^{-2} = \pm 8\ \%$

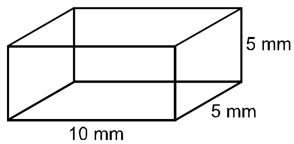
Q.31 (2)

Δt = 0.2 s.

t = 25 s

$$T = \frac{t}{N} \Rightarrow \frac{\Delta T}{T} = \frac{\Delta t}{t} = \frac{0.2}{25} = 0.8\ \%$$

Q.32 (1)



$v = \ell bh$

$$\frac{\Delta v}{v} = \frac{\Delta \ell}{\ell} + \frac{\Delta b}{b} + \frac{\Delta h}{h}$$

$$= \frac{0.1}{10} + \frac{0.1}{5} + \frac{0.1}{5} = \frac{0.5}{10} = \pm 5\%$$

Q.33 (4)

$$\frac{\Delta x}{x} = 1\% = 10^{-2}$$

$$\frac{\Delta y}{y} = 3\% = 3 \times 10^{-2}$$

$$\frac{\Delta z}{z} = 2\% = 2 \times 10^{-2}$$

$$t = \frac{xy^2}{z^3}$$

$$\frac{\Delta t}{t} = \frac{\Delta x}{x} + \frac{2\Delta y}{y} + \frac{3\Delta z}{z}$$

$= 10^{-2} + 2 \times 3 \times 10^{-2} + 3 \times 2 \times 10^{-2}$

$= 13 \times 10^{-2}$ $\therefore$ % error in t $= \frac{\Delta t}{t} \times 100 = 13\%$

Q.34 (3)

D = (4.23 ± 0.01) cm

d = (3.89 ± 0.01) cm

Δt = (D – d)/2

$$= \frac{(4.23 \pm 0.01) - (3.89 \pm 0.01)}{2}$$

$$= \frac{(4.23 - 3.89) \pm (0.01 + 0.01)}{2}$$

= (0.34 ± 0.02)/2 cm

= (0.17 ± 0.01)cm

Q.35 (2)

m = 1.76 kg

M = 25 m

= 25 × 1.76

= 44.0 kg

Note : Mass of one unit has three significant figures and it is just multiplied by a pure number (magnified). So result should also have three significant figures.

Q.36 (2)

$R_1 = (24 \pm 0.5)\ \Omega$

$R_2 = (8 \pm 0.3)\ \Omega$

$R_S = R_1 + R_2$

$= (32 \pm 0.8)\ \Omega$

Q.37 (2)

$\Delta \ell = 0.5$ mm

N = 100 divisions

zero correction = 2 divisions

Reading = Measured value + zero correction

$= (8 \times 0.5)$ mm $+ (83 - 2) \times \frac{0.5}{100}$.

$= 4$ mm $+ 81 \times \frac{0.5}{100}$ mm

= 4.405 mm

Q.38 (4)

$\Delta \ell = 1$ mm

N = 50 division

zero error = –6 Divisions

= – 0.12 mm

Diameter = Measured value + zero correction

$= 3 \times 1 + (6 + 31) \times \frac{1}{50}$

= 3 + 0.74 = 3.74 mm

Q.39 (1)

$D = 2 \times 1 + 5 \times \frac{10 - 9}{100} = 2.05$ cm

Q.40 (2)

Volume of a sphere $= \frac{4}{3}\pi\ (\text{radius})^3$

or $V = \frac{4}{3}\pi R^3$

Taking logarithm on both sides, we have

$\log V = \log \frac{4}{3}\pi + 3 \log R$

Differentiating, we get

$$\frac{\Delta V}{V} = 0 + \frac{3\Delta R}{R}$$

Accordingly, $\frac{\Delta R}{R} = 2\%$

Thus, $\frac{\Delta V}{V} = 3 \times 2\% = 6\%$

JEE ADVANCED
OBJECTIVE QUESTIONS

Q.1 (B)
Solar day $\rightarrow$ Time far Earth to wake a complete rotation on its axis
Parallactic second [1 Parsec] $\rightarrow$ It is a distance corresponding to a parallex of one second of arc.
Leap year $\rightarrow$ A leap year is year (time) Containing one extra day.
Lunar Month $\rightarrow$ A lunar month is the time between two identical view moons of full moons.
1 Lunar month = 29.53059 days.

Q.2 (B)
Unit of impulse = $\Rightarrow$ Impulse = Force × time

$$= kg\ \frac{m}{sec^2} sec\ = kg \frac{m}{sec}\ = mv$$

The unit is same as the unit of linear momentum.

Q.3 (D)
Energy W = f × d = Nm
W = eV = electron-volt
W = p × t = Watt hour
So, kg × m/sec^2 is not the unit of energy.

Q.4 (D)
Radian is a unit of Angle.

Q.5 (C)
Dimensionless quantity may have a unit
Ex. Angle Unit $\rightarrow$ Radian
Dimension $\rightarrow$ $M^oL^oT^o$

Q.6 (C)
Only same physical quantities can be added or substracted,
It's only multiply and divided only.
So, a/b denote a new physical quantity.

Q.7 (C)
They Can't e added or Substracted in Same expression.

Q.8 (D)
Plank Const. (h) $\rightarrow$
E = hf

$$\Rightarrow \quad h = \frac{ML^2\ T^{-2}}{\frac{1}{T}}$$

Unit $\rightarrow$ J-S
Dimension = $M^1L^2T^{-2} \times T^1 = M^1L^2T^{-1}$

This is also a dimension of Angular momentum.
= mvr
= MLT^{-1} $L=M^1L^2T^{-1}$

Q.9 (B)
$P = P_o$ Exp $(-\alpha t^2)$
Here Exp $(-\alpha t^2)$ is a dimensionless
So, dimension of $[\alpha t^2] = M^oL^oT^o$

$$\text{So, } [\alpha] = \frac{M^oL^oT^o}{T^2}$$

$[\alpha] = M^oL^oT^{-2}$

Q.10 (C)
By Checking the dimension in all options
(C) Moment of Inertia = Mr^2
$= M^1L^2T^o$
Moment of force = r × F
$= L^1 \times M^1L^1T^{-2}$
$= M^1L^2T^{-2}$

Q.11 (D)
Action = Energy × Time = $M^1L^2T^{-2} \times T^1$
$= M^1L^2T^{-1}$
It is same as dimension of Impulse × distance
$= MLT^{-1} \times L^1 = M^1L^2T^{-1}$

Q.12 (A)
$M^1L^2T^{-2}$ is a dimension of kinetic energy.

Q.13 (C)
By checking the dimension in all options
[Pressure] = $M^1L^{-1}T^{-2}$

Q.14 (B)

$$\frac{EJ^2}{M^5G^2}\ \ J=mvr,\ J = [ML^2T^{-1}]$$

$= [M^0L^0T^0]$
Dimension of Angle = $[M^0L^0T^0]$

Q.15 (C)

$$v = at + \frac{b}{t+c}$$

Same physical quantity can be added or substracted.
Dimension of a
$[v] = [at]$

$$[a] = \frac{[v]}{[t]} = \frac{L^1T^{-1}}{T^1} = L^1T^{-2}$$

Here t + c is also a Time (t)

$$[v] = \left[\frac{b}{t}\right]$$

$[b] = [v]\ [t] = L^1T^{-1} \times T^1$

$[b] = L^1$

Q.16 (C)

$$x(t) = \frac{v_o}{\alpha}[1 - e^{-\alpha t}]$$

Dimension of v_o and α

Here $e^{-\alpha t}$ is dimensionless so,

$[\alpha]\,[t] = M^oL^oT^o$

$$[\alpha] = \frac{M^oL^oT^o}{T^1} = T^{-1}$$

$[\alpha] = M^oL^oT^{-1}$

Here $1-e^{-\alpha t}$ is a number

$$[x(t)] = \frac{V_o}{\alpha}$$

$[V_o] = [L^1]\,[T^{-1}]$

$[V_o] = M^oL^1T^{-1}$

Q.17 (D)

$F = Pt^{-1} + \alpha t$

Here F and Pt^{-1} is a same

Physical quantity

$[F] = [Pt^{-1}]$

$$[P] = \frac{[F]}{[t^-]} = [F \times t] = ML\,T^{-2} \times T = MLT^{-1}$$

We find it is same as dimension of momentum $= MLT^{-1}$

Q.18 (D)

$Y = a \sin (bt - cx)$

Dimension of b

Here bt is dimensionless

$[bt] = M^oL^oT^o$

$$[b] = \frac{M^oL^oT^o}{[T^1]} = M^oL^oT^{-1}$$

It is a dimension of wave frequency.

Q.19 (A)

Dimension of b $= M^oL^oT^{-1}$

Dimension of c

$[cx] = M^oL^oT^o$ [Dimension less]

$$[c] = \frac{M^oL^oT^o}{L^1} = M^oL^{-1}T^o$$

So, Dimension of $\left[\frac{b}{c}\right] = \frac{M^oL^oT^{-1}}{M^oL^{-1}T^o} = M^oL^1T^{-}$

Q.20 (C)

Here $\sqrt{1 + \frac{2k\ell}{ma}}$ is a number.

It's a dimensionless quantity.

$$\left[\frac{2k\ell}{ma}\right] = [M^oL^oT^o]$$

$$[K] = \frac{[m][a]}{[\ell]}$$

$$= \frac{M^1L^1T^{-2}}{L^1} = M^1L^oT^{-2}$$

So dimession of [b] is

$$[b] = \left[\frac{ma}{K}\right] = \left[\frac{MLT^{-2}}{MT^{-2}}\right]$$

$[b] = L$

unit of b is metre

Q.21 (D)

$$\alpha = \frac{F}{V^2}\sin(\beta t)$$

Here sin (βt) is dimensionless.

$[\beta t] = M^oL^oT^o$

$$\beta = \frac{M^oL^oT^o}{T^1} = \left[T^{-1}\right]$$

$$[\alpha] = \left[\frac{F}{V^2}\right]$$

$$= \frac{M^1L^1T^{-2}}{[L^1T^{-1}]^2} = \frac{M^1L^1T^{-2}}{L^2T^{-2}}$$

$[\alpha] = [M^1L^{-1}T^o]$

Q.22 (D)

$L \,\alpha\, FAT$

$L = K\,F^aA^bT^c$ .... (i)

$M^oL^1T^o = K[M^1L^1T^{-2}]\,[L^1T^{-2}]^b\,[T]^c$

$M^oL^1T^o = K[M^a]\,[L^{a+b}]\,[T^{-2a-2b+c}]$

By comparesion and solving we find

$[a = 0]\ [b = 1]\ [c = 2]$

Put these value in Equa. (i)

$[L = F^oA^1T^2]$

Q.23 (B)

$F \,\alpha\, Av\rho$

$F = KA^av^b\rho^c$

$= K[L^2]^a\,[L^1T^{-1}]^b\,[M^1L^{-3}]^c$

$F = K[M^cL^{2a+b-3c}\,T^{-b}]$

$M^1L^1T^{-2} = K[M^c\,L^{2a+b-3c}\,T^{-b}]$

$c = 1$

$-2 = -b \Rightarrow b = 2$

and

$2a + b - 3c = 1$

$2a + 2 - 3 = 1 \Rightarrow a = 1$

So $F = A^1 v^2 q^1$

$\therefore F = Av^2\rho$

Q.24 (B)

$v \propto \lambda\rho g$

$v = k\lambda^a \rho^b g^c$

$[M^oL^1T^{-1}] = K[M^oL^1T^o]^a[M^1L^{-3}T^o]^b[M^oL^1T^{-2}]^c$

$[M^oL^1T^{-1}] = K[M^bL^{a-3b+c}T^{-2c}]$

Comparing both sides

$b = 0$

$-1 = -2c \Rightarrow c = \frac{1}{2}$

$1 = a - 3b + c$

$1 = a - 0 + 1/2$

$a = \frac{1}{2}$, $V = K\lambda^{1/2}q^og^{1/2}$

squaring both sides

$V^2 = kg\lambda$

Q.25 (B)

$F = KAdv^x$

$M^1L^1T^{-2} = K[L^2]\,[M^1L^{-3}]\,[L^1T^{-1}]^x$

$M^1L^1T^{-2} = K[M^1L^{-1+x}T^{-x}]$

By comparison of power

$-1 + x = 1 \Rightarrow x = 2$

Q.26 (B)

$V = g^p h^q$

$V = Kg^p h^q$

$[L^1T^{-1}] = [L^1T^{-2}]^p [L^1]^q$

$L^1T^{-1} = L^{p+q}\,T^{-2p}$

By comparing both sides

$p+q=1,\ -2p=-1$

$p = 1/2,\ q=1/2$

Q.27 (D)

$\ell \alpha c^a g^b p^c$

$\ell = kc^a g^b p^c$

$[M^0L^1T^0] = K[L^1T^{-1}]^a\,[L^1T^{-2}]^b\,[M^1L^{-1}T^{-2}]^c$

$M^0L^1T^0 = K[M^cL^{a+b-c}T^{-a-2b-2c}]$

By comparison of powers

$a = 2$

$b = -1$

$c = 0$

So, $\ell = c^2/g$

Q.28 (D)

Unit of length is micrometer

Unit of time is mirosecond

$\because$ Velocity $= \frac{\text{Displacement}}{\text{Time taken}}$

$= \frac{10^{-6}\,m}{10^{-6}\,sec} = m/sec$

Q.29 (A)

$n_1u_1 = n_1u_1$

$n_1\left[M_1^1L_1^2T_1^{-3}\right] = 1\left[M_2^1L_2^2T_2^{-3}\right]$

$n_1 = \left[\frac{M_2}{M_1}\right]^1\left[\frac{L_2}{L_1}\right]^2\left[\frac{T_2}{T_1}\right]^{-3}$

$= \left[\frac{20}{1}\right]^1\left[\frac{10}{1}\right]^2\left[\frac{5}{1}\right]^{-3}$

$= \frac{20\times100}{5\times5\times5} = 16$

$n_1 = 16$

Unit of power in new system = 16 Watt.

Q.30 (C)

$10^3(N) = M^1L^1T^{-2}$

$10^3 = [M]^1\,[10^3]^1\,[100]^{-2}$

$M = \frac{10^3}{10^3 \times (100)^{-2}} = 10000\,kg$

Q.31 (D)

$g = 10\ ms^{-2}$

$n_1u_1 = n_2u_2$

$10[L_1]^1[T_1]^{-2} = n_2[L_2]^1[T_2]^{-2}$

$n_2 = 10\left[\frac{L_1}{L_2}\right]^1\left[\frac{T_1}{T_2}\right]^{-2}$

$n_2 = 10\left[\frac{1}{1000}\right]^1\left[\frac{1}{3600}\right]^{-2}$

$n_2 = 129600$

Q.32 (C)

In new system

Length $\rightarrow$ m 2m

Velocity $\rightarrow$ m/sec. 2m/sec

Force $\rightarrow$ kgm/sec^2 2kgm/sec^2

$\because$ Momentum (P) = mv = kg m/sec.

$P = kg\frac{m}{sec}\times\frac{m}{m}\times\frac{sec}{sec}$

$$P = kg\frac{m}{sec^2}\times\frac{m}{(m/sec)}$$

In new system

$$P^1 = \left(2kg\frac{m}{sec^2}\right)\times\frac{(2m)}{(2m/sec)}$$

$$P^1 = (2kg\,m/sec) = 2P$$

So, Here unit of momentum is doubled.

Q.33 (D)

Unit of Energy = $kg\frac{m^2}{sec^2}$

$$= \left(kg\frac{m}{sec^2}\right)\times(m)$$

Now unit of force and length are doubled.

$= \left(2kg\frac{m}{sec^2}\right)(2m) \quad = 4kg\frac{m^2}{sec^2}$ So, Unit of Energy is 4 times.

Q.34 (C)

$$K.E. = \frac{1}{2}mv^2$$

Dimension = $M^1L^2T^{-2}$

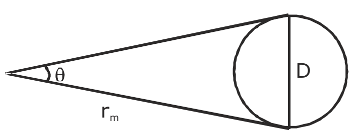
Now M.L are doubled

= $(2M)^1 (2L)^2 (T^{-2}) = 8\, M^1L^2T^{-2}$

So, K.E. will become 8 times.

Q.35 (B)

Take small angle approximation



$$\text{Sin } \theta = \frac{D}{r_m}$$

$$\text{Sin } 0.50° = \frac{D}{r_m}$$

$$0.50\times\frac{\pi}{180} = \frac{D}{384000}$$

$$D = 0.50\times\frac{\pi}{180}\times 384000$$

D = 3349.33 $\Rightarrow$ D $\approx$ 3350 km.

JEE-ADVANCED

MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 (A), (B), (C), (D)

All A,B & C are obvious.

Q.2 (A), (B), (D)

It is obvious

Q.3 (C), (D)

By checking dimension in each option

$$\text{Pressure } = \frac{F}{A} = \frac{[M^1L^1T^{-2}]}{[L^2]} = [M^1L^{-1}T^{-2}]$$

$$\text{Energy per unit volume } = \frac{\text{Energy}}{\text{Volume}} = \frac{[M^1L^2T^{-2}]}{[L^3]} = [M^1L^{-1}T^{-2}]$$

Q.4 (A), (B), (C)

$$[\alpha] = \frac{[h]}{[\sigma\theta^4]} = \frac{ML^2T^{-1}}{MT^{-3}K^{-4}.K^4} = L^2T^2$$

So, unit of α will be m^2s^2.

$$\frac{(weber)(\Omega)^2(Farad)^2}{Tesla} = \frac{Tm^2.\Omega^2F^2}{T} = m^2s^2$$

Q.5 (B)

Q.6 (A)

Q.7 (C)

$[f] = [A]\,[t]$

$= [B]\,[t]^2$

$= [C]\left[\frac{1}{t}\right]$

Q.8 (C)

Q.9 (D)

Q.10 (B)

$[C] = [F]\,[r]^2 = ML^3T^{-2}$

$[kr] = 1$

$\Rightarrow [k] = L^{-1}$

Q.11 (B)

Q.12 (C)

Q.13 (D)

$m = h^xc^yG^z$

$\Rightarrow M = (ML^2T^{-1})^x (LT^{-1})^y (M^{-1}L^3T^{-2})^z$

$l = h^xc^yG^z \quad \Rightarrow L = (ML^2T^{-1})^x (LT^{-1})^y(M^{-1}L^3T^{-2})^z$

$t = h^xc^yG^z \quad \Rightarrow T = (ML^2T^{-1})^x (LT^{-1})^y (M^{-1}L^3T^{-2})^z$

Q.14 **(1) → (Q) → (c) ,(2) → (S) → (a), (3) → (P) → (b), (4) → (R) → (d)**

$F = G\frac{m_1m_2}{r^2} \Rightarrow [G] = \frac{[F][r^2]}{[m_1m_2]} = \frac{MLT^{-2}L^2}{M^2} = M^{-1}L^3T^{-2}$

$[\text{Torque}] = [f]\,[d] = MLT^{-2}L = ML^2T^{-2}$

$[\text{Momentum}] = [m]\,[v] = MLT^{-1}$

$[p] = \frac{[F]}{[A]} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$.

Q.15 **(i) → (Q) → (d), (ii) → (S) → (b), (iii) → (U) → (e), (iv) → (R) → (a), (v) → (T) → (f) (vi) → (P) → (c)**

(i) $U = \sigma AT^4 \Rightarrow [\sigma] = \frac{[U]}{[A][T^4]} = \frac{ML^2T^{-3}}{L^2K^4}$

$= MT^{-3}K^{-4}$

(ii) $\lambda T = b \Rightarrow [b] = [\lambda]\,[T] = LK$

(iii) $F = 6\pi\eta rv \Rightarrow [\eta] = \frac{[F]}{[r][v]} = \frac{MLT^{-2}}{L.\,LT^{-1}} = ML^{-1}T^{-1}$

(iv) $I = \frac{P}{A} = \frac{ML^2T^{-3}}{L^2} = ML^oT^{-3}$

(v) Energy $= \frac{1}{2}Mi^2 \Rightarrow [M] = \frac{[E]}{[i^2]} = ML^2T^{-2}A^{-2}$

(vi) $\frac{[U]}{[V]} = \frac{[B^2]}{[2\mu_0]} = [\mu_0] = \frac{[B^2]\,[V]}{[U]}$

Also , $F = qVB \Rightarrow B = \frac{F}{qv}$

$[\mu_0] = \frac{(F)^2[V]}{[q^2v^2][U]} = MLT^{-2}A^{-2}$ **Ans.**

NUMERICAL VALUE BASED

Q.1 625

$[E] = mL^2T^{-2}$

$$K_2 = K_1\left[\frac{m_1}{m_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

$$= 10\left[\frac{1\text{kg}}{100\text{gm}}\right]^1\left[\frac{1\text{m}}{200\text{cm}}\right]^2\left[\frac{1\text{sec}}{5\text{sec}}\right]^2$$

$$= 10\left[\frac{1000\text{gm}}{100\text{gm}}\right]\left[\frac{100\,\text{cm}}{200\,\text{cm}}\right]^2[5]^2$$

$$= 10 \times 10 \times \frac{1}{4} \times 25 = 625 \text{ Ans.}$$

Q.2 4

$F \propto v^a$

$\propto \rho^b$

$\propto A^c$

$\Rightarrow F = k\, v^a \rho^b A^c$ k : dimensional constant.

By dimension analysis a = 2 $\Rightarrow F \propto v^2$.

Q.3 0008

$P = k\,\omega^a \rho^b \ell^c$

$\Rightarrow ML^2T^{-3} = (T^{-1})^a (ML^{-3})^b L^c$

$\Rightarrow a = 3$

$\therefore P \propto \omega^3$

Q.4 0002

$\left[\frac{at^x}{A}\right] = 1 \Rightarrow \frac{[a][t]^x}{[s]} = 1 \Rightarrow x = 2$

Q.5 [3]

$[V] = \frac{ML^2T^{-2}}{AT} = ML^2T^{-3}A^{-1}$

KVPY PREVIOUS YEAR'S

Q.1 (C)

$\sigma = h^\alpha k_B^\beta c^\gamma$

$\sigma \to$ Stefan boltzmann constant

$h \to$ Planck's constant

$k_B \to$ Boltzmann constant

$c \to$ speed of light

According to stefan's law

$\frac{Q}{At} = \sigma T^4$

$\sigma = \frac{Q}{At} \times \frac{1}{T^4}$

$[\sigma] = \frac{[M^1L^2T^{-2}]}{[L^2][T][K^4]} = [M^1T^{-3}K^{-4}]$

& $E = h\nu$

$\Rightarrow h = \frac{E}{\nu} \Rightarrow [h] = \frac{[M^1L^2T^{-2}]}{[T^{-1}]}$

$[h] = [M^1L^2T^{-1}]$

$[c] = [LT^{-1}]$

$E = \frac{3}{2}k_BT$

$\Rightarrow \quad [k_B] = \dfrac{[M^1L^2T^{-2}]}{[K]}$

$[k_B] = [M^1L^2T^{-2}K^{-1}]$

According to homogeneity principle of dimension

$[M^1T^{-3}K^{-4}] = [M^1L^2T^{-1}]^{\alpha}[M^1L^2T^{-2}K^{-1}]^{\beta}[L^1T^{-1}]^{\gamma}$

on comparing powers of M, L, T, K on both sides

$\Rightarrow \quad \alpha + \beta = 1 \quad ...(1)$
$2\alpha + 2\beta + \gamma = 0 \quad ....(2)$
$-\alpha - 2\beta - \gamma = -3 \quad ...(3)$
$-\beta = -4 \quad ....(4)$
$\Rightarrow \quad \beta = 4$
$\alpha = -3$

Putting values is equation (2)
$2(-3) + 2(4) + \gamma = 0$
$\gamma = -2$

Ans.
$\alpha = -3$
$\beta = 4$
$\gamma = -2$

Q.2 (1)

$$\frac{V}{t} = k\left(\frac{p}{\ell}\right)^a \eta^b r^c$$

$V \rightarrow$ volume
$P \rightarrow$ pressure
$\eta \rightarrow$ coefficient of viscosity
$r \rightarrow$ radius

Using dimensional analysis
$[M^0L^3T^{-1}] = [M^1L^{-2}T^{-2}]^a[M^1L^{-1}T^{-1}]^b[L]^c$
$[M^0L^3T^{-1}] = [M^{a+b}L^{-2a-b+c}T^{-2a-b}]$
$a + b = 0 \quad ...(1)$
$-2a - b + c = 3 \quad ....(2)$
$a = -b$
$-2a - b = -1 \quad ...(3)$
put the value of $-2a - b = -1$ in equation (2)
$-1 + c = 3$
$c = 4$
put $a = -b$ in equaiton (3)
$2b - b = -b$ in equation (3)
and $a = 1$
hence option (A) is correct.

Q.3 (B)
$A = G^{\alpha}M^{\beta}C^{\gamma}$
$[M^0L^2T^0] = [M^{-1}L^3T^{-2}]^{\alpha}[M]^{\beta}[LT^{-1}]^{\gamma}$
$-\alpha + \beta = 0 \Rightarrow \alpha = \beta$
$3\alpha + \gamma = 2$
$-2\alpha - \gamma = 2$
on solving
$\alpha = 2, \beta = 2, \gamma = -4$

Q.4 (A)
$P = [ML^2T^{-3}]$
$\mu_0 = [MLT^{-2}T^{-2}]$
$m = [IL^2]$
$\omega = [T^{-1}]$
$C = [LT^{-1}]$
$\therefore [ML^2T^{-3}] = [MLT^{-2}I^{-2}]^x[IL^2]^y[T^{-1}]^z[LT^{-1}]^u$
$x = 1, y = 2, z = 4, u = -3$

Q.5 (C)

$$\rho = \frac{E}{J}$$

$E = [MLT^3I^{-1}]$
$J = [IL^{-2}]$
$h = ML^2T^{-1}$
$m_e = M$
$c = LT^{-1}$
$\varepsilon_0 = M^{-1}L^{-3}T^4I^2$

$$\therefore \rho = \frac{h^2}{m_e c e^2}$$

Q.6 (B)
$1° = 3600$ arc sec
average change in orientation per year

$\dfrac{180^o}{10^5}$ degree/year

$\dfrac{180 \times 3600}{10^5}$ sec/year

$= 1.8 \times 0.36$
$= 6.48$ sec/year
Closest option (B)

Q.7 (D)
$[T'] = [m^aL^{-a}T^{-2a} \quad m^bL^{-3b}m^cT^{-2c}]$
$[T'] = [m^{a+b+c}L^{-a-3b}T^{-2a-2c}]$
$a + b + c = 0$
$-a - 3b = 0$
$-2a - 2c = 1$
On solving

$$a = \frac{-3}{2}, b = \frac{1}{2}, c - 1$$

Q.8 (A)
Precession mean
every time reading is coming nearly same.

Q.9 (C)
Force $F = 10.0 \pm 0.2$ N
$a = 1.00 \pm 0.01$ m/s^2

$$F = ma \Rightarrow m = \frac{F}{a}$$

$$m = \frac{10.0}{1.00}$$

m = 10.0 kg

For error (F = ma)

$m^1 a^1 F^{-1}$ = const.

$$\frac{dm}{m}+\frac{da}{a}-\frac{dF}{F}=0$$ [Take log and differentiate]

$$\frac{\Delta m}{m}=\left|\frac{\Delta F}{F}-\frac{\Delta a}{a}\right|_{max}$$

$$\frac{\Delta m}{m}=\left|\frac{0.2}{10.0}+\frac{0.01}{1.00}\right|$$

$$\Delta m=\frac{3}{100}m$$

$$\Delta m=\frac{3}{100}\times 10kg$$

Δm = 0.3 kg

mass m = (10.0 ± 0.3kg)

Q.10 (C)

$A = \ell B$

$dA = \ell\, dB + B\, d\ell$

$$\frac{dA}{A}=\frac{\ell dB}{\ell B}+\frac{Bd\ell}{\ell B}$$

$$\frac{dA}{A}=\frac{dB}{B}+\frac{d\ell}{\ell}$$

$$=\frac{0.05}{3.05}+\frac{0.05}{3.95}$$

= 0.016 + 0.012

= 0.028 × 12.05

dA = 0.33

12.05 ± 0.34

Q.11 (D)

x = 0.3 cm – 3 × scale division of vernier calipers

$$= 0.3\text{ cm} - 3\times\frac{9}{100}$$

$$=\frac{30-27}{100}$$

$$=\frac{3}{100}$$

= 0.03 cm

Q.12 (A)

In vernier Callipers, given that

10 V.S.D. = 11 mm

$$1\text{ V.S.D.} = \frac{11}{10}\text{ mm} = 1.1\text{ mm}$$

L.C. = | 1 M.S.D. – 1 V.S.D. |.

Magnitude of L.C. = 0.1 mm

Q.13 (A)

Student A : Length of scale = 25 m

Least count = 0.5 cm = 0.005m

Student A can measure the length of 9.5m by using the scale only once so there will be an error of 0.005 m in 9.5 m

$\therefore$ Relative error = $\frac{0.005}{9.5}$ = 0.005

Student B : Length of scale : 1m = 100 cm

Least count = 0.05 cm

To measure 9.5m, student B has to use this meter scale atleast 10 times

$\therefore$ Relative error = $\frac{0.05}{100}$ × 10 = 0.005

Student C : Length of scale : 1 foot = 30.48 cm

Least count = 0.05 cm

To measure 9.5m, student C has to use this scale approximately 31 times

$\therefore$ Relative error = $\frac{0.05}{30.48}$ × 31 = 0.05 cm

$\therefore$ Relative error is least for Student A.

JEE-MAIN
PREVIOUS YEAR'S

Q.1 (1)

$KE = M^1L^2T^{-2}$

$P = M^1L^1T^{-1}$

$h = M^1L^2T^{-1}$

$v = M^1L^2T^{-2}C^{-1}$

Q.2 (4)

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{hc} = \frac{Ke^2\times\lambda^2}{\lambda^2\times hc} = \frac{F\times\lambda}{E} = \frac{E}{E}$$; dimension less

Q.3 (2)

$\frac{\beta x^2}{KT}$ is dimension less so $KT = \beta x^2$

$\Rightarrow M^1L^2T^{-2} = \beta L^2$

$\beta = M^1 T^{-2}$

$w = \alpha^2\times\beta$

$M^1 L^2 T^{-2} = \alpha^2 \times M^1 T^{-2}$

$\alpha = L$

Q.4 (4)

$$[\lambda] = \frac{[C]}{[V]} = \frac{[M^{-1}L^{-2}T^4A^2]}{[M^2LT^{-3}A^{-1}]}$$

$$= [M^{-3}L^{-3}T^7A^3]$$

Q.5 (1)

$$\frac{x^2}{\alpha KT} = \text{dimensionless}$$

$$\frac{L^2}{KT} \Rightarrow \alpha$$

$$\alpha \Rightarrow \frac{L^2}{V^2M} = L^2M^{-1}L^{-2}T^2 = M^{-1}T^2$$

work = $\alpha \cdot \beta^2 \cdot$ (dimensionless)

$M^1L^1T^{-2} \cdot L^1 = M^{-1}T^2\beta^2$

$$v = \sqrt{\frac{3KT}{M}}$$

$$\frac{v^2 \cdot M}{3} = KT$$

$\beta = M^1L^1T^{-2}$

Q.6 [25]

$$4tT = 100 \times \frac{4}{3}\pi r^3$$

$$= 100 \times \frac{4\pi}{3} \times \frac{3}{40\pi} \times 10^{-9} = 10^{-8}\ cm^3$$

$t_T = 25 \times 10^{-10}$ cm

$= 25 \times 10^{-12}$ m

$t_0 = 0.01\ t_T = 25 \times 10^{-14}$ m

= 25

Q.7 (4)

Q.8 (4)

Q.9 [13]

Q.10 (2)

Q.11 (1)

Q.12 (1)

Q.13 (1)

Q.14 (2)

Q.15 (4)

$E = M L^2 T^{-2}$

$L = M L^2 T^{-1}$

m = M

$G = M^{-1} L^{+3} T^{-2}$

$$P = \frac{EL^2}{M^5G^2}$$

$$[P] = \frac{(ML^2T^{-2})(M^2L^4T^{-2})}{M^5(M^{-2}L^6T^{-4})} = M^0L^0T^0$$

Q.16 (3)

$$\text{Least count (L.C)} = \frac{0.5}{50}$$

$$\text{True reading} = 5 + \frac{0.5}{50} \times 20 - \frac{0.5}{50} \times 5$$

$$= 5 + \frac{0.5}{50}(15) = 5.15\text{mm}$$

Option (3)

Q.17 (1)

Q.18 [52]

9 MSD = 10 VSD

9 × 1 mm = 10 VSD

∴ 1 VSD = 0.9 mm

LC = 1 MSD –1 VSD = 0.1 mm

Reading = MSR + VSR × LC

10 + 8 × 0.1 = 10.8 mm

Actual reading = 10.8 – 0.4 = 10.4 mm

$$\text{radius} = \frac{d}{2} = \frac{10.4}{2} = 5.2\text{ mm}$$

$= 52 \times 10^{-2}$ cm

Q.19 (2)

$[M] = K[F]^a [T]^b [V]^c$

$[M^1] = [M^1L^1T^{-2}]^a [T^1]^b [L^1T^{-1}]^c$

a = 1, b = 1, c = $^{-1}$

∴ $[M] = [FTV^{-1}]$

Q.20 (1)

Q.21 (3)

Q.22 (1)

Q.23 (3)

$$\text{least count} = \frac{1}{100}\text{mm.}$$

+ve error = +0.08 mm.

Measured reading (Diameter)

$$= 1\text{mm} + \left(72 \times \frac{1}{100}\right)\text{mm}$$

Original (True reading) = 1.72 – 0.08 = 1.64 mm

So original radius = 0.82 mm.

Q.24 (3)

$T^2 = 4\pi^2\left[\frac{\ell}{g}\right]$

$g = 4\pi^2\left[\frac{\ell}{g}\right]$

$\frac{\Delta g}{g} = \frac{\Delta\ell}{\ell} + \frac{2\Delta T}{T}$

$= \left[\frac{1mm}{1m} + \frac{2(10\times10^{-3})}{1.95}\right]\times100$

$= 1.12\%$

Q.25 (4)

$(n-1)a = n(a')$

$a' = \frac{(n-1)a}{n}$

$\therefore$ L.C. = 1 MSD – 1 VSD

= (a – a')cm

$a' - \frac{(n-1)a}{n}$

$= \frac{na - na + a}{n} = \frac{a}{n}cm$

$\left(\frac{10a}{n}\right)mm$

Q.26 [5]

$\frac{\Lambda R}{R}\times100 = \frac{\Lambda V}{V} = 100 + \frac{\Lambda I}{I}\times100$

% error in R = $\frac{2}{50}\times100 + \frac{0.2}{20}\times100$

% error in R = 4 + 1

% error in R = 5%

Q.27 (4)

$Y = \frac{Stress}{Strain} = \frac{FL}{Al} = \frac{mg.L}{\pi R^2.\ell}$

$\frac{\Delta Y}{Y} = \frac{\Delta m}{m} + \frac{\Delta L}{L} + 2.\frac{\Delta R}{R} + \frac{\Delta\ell}{\ell}$

$\frac{\Delta Y}{Y}\times100 = 100\left[\frac{1}{1000} + \frac{1}{1000} + 2\left(\frac{0.001}{0.2}\right) + \frac{0.001}{0.5}\right]$

$= \frac{1}{10} + \frac{1}{10} + 1 + \frac{1}{5} = \frac{14}{10} = 1.4\%$

Q.28 (2)

Positive zero error = 0.2 mm

Main scale reading = 8.5 cm

Vernier scale reading = 6 × 0.01 = 0.06 cm

Final reading = 8.5 + 0.06 – 0.02 = 8.54 cm

Q.29 (2)

$\gamma = \frac{4\pi^2\ell}{T^2}$

$\frac{\Delta g}{g} = \frac{\Delta\ell}{\ell} + 2\frac{\Delta T}{T} = \frac{0.1}{10} + 2\left(\frac{\frac{1}{200}}{0.5}\right)$

$\frac{\Delta g}{g} = \frac{1}{100} + \frac{1}{50}$

$\frac{\Delta g}{g}\times100 = 3\%$

Q.30 (1)

$R = \frac{\rho\ell}{A} = \frac{V}{I}$

$\rho = \frac{AV}{I\ell} = \frac{\pi d^2 V}{4I\ell}$ $\quad\left(A = \frac{\pi d^2}{4}\right)$

$\therefore \frac{\Delta\rho}{\rho} = \frac{2\Delta d}{d} + \frac{\Delta V}{V} + \frac{\Delta I}{I} + \frac{\Delta\ell}{\ell}$

$\frac{\Delta\rho}{\rho} = 2\left(\frac{0.01}{5.00}\right) + \frac{0.1}{5.0} + \frac{0.01}{2.00} + \frac{0.1}{10.0}$

$\frac{\Delta\rho}{\rho} = 0.004 + 0.02 + 0.005 + 0.01$

$\frac{\Delta\rho}{\rho} = 0.039$

% error = $\frac{\Delta\rho}{\rho}\times100 = 0.039\times100 = 3.90\%$

Q.31 [34]

Q $v = \frac{4}{3}\pi r^3$

taking log & then differentiate

$\frac{dV}{V} = 3\frac{dr}{r}$

$= \frac{3\times0.85}{7.5}$´ 100% = 34 %

Q.32 (1)

Q.33 (1)

Q.34 (3)

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{4} + \frac{1}{4} \Rightarrow R_{eq} = 2\Omega$$

Also $\frac{\Delta R_{eq}}{R_{eq}^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$

$$\frac{\Delta R_{eq}}{4} = \frac{.8}{16} + \frac{.4}{16} = \frac{1.2}{16}$$

$\triangleq R_{eq} = 0.3\Omega$

$R_{eq} = (2 \pm 0.3)\Omega$

Q.35 [14]

Q.36 (3)

JEE-ADVANCED PREVOUS YEAR'S

Q.1 (C)

(p) $U = \frac{1}{2}kT \Rightarrow ML^2T^{-2} = [k]\, K \Rightarrow [K] = ML^2T^{-2}K^{-1}$

(q) $F = \eta A\frac{dv}{dx} \Rightarrow [\eta] = \frac{MLT^{-2}}{L^2LT^{-1}L^{-1}} = ML^{-1}\, T^{-1}$

(r) $E = h\nu \Rightarrow ML^2T^2 = [h]\, T^{-1} \Rightarrow [h] = ML^2\, T^{-1}$

(s) $\frac{dQ}{dt} = \frac{kA\Delta\theta}{\ell} \Rightarrow [k] = \frac{ML^2T^{-3}L}{L^2K} = MLT^{-3}\, K^{-1}$

Q.2 [3]

$d = k\,(\rho)^a\,(S)^b\,(f)^c$

$$\left[\frac{M}{L^3}\right]^a \left[\frac{M^1L^2T^{-2}}{L^2\,T}\right]^b \left[\frac{1}{T}\right]^c$$

$0 = a + b$

$1 = -3a \Rightarrow a = -\frac{1}{3}$

So $b = \frac{1}{3}$ $\Rightarrow n = 3$

Q.3 (B,D)

$$[k_BT] = [Fl]$$

$$\& \left[\frac{q^2}{\in}\right] = [Fl^2]$$

Q.4 (C)

We have $\frac{E}{C} = B$

$\therefore [B] = \frac{[E]}{[C]} = [E]L^{-1}T^1 \quad \Rightarrow [E] = [B]\,[L]\,[T]^{-1}$

Q.5 (D)

We have $C = \frac{1}{\sqrt{\mu_0 \in_0}} \therefore [C^2] = \left[\frac{1}{\mu_0 \in_0}\right]$

$\Rightarrow L^2T^{-2} = \frac{1}{[\mu_0][\in_0]}$

$\Rightarrow [\mu_0] = [\mu_0] = [\in_0]^{-1}\,[L]^{-2}\,[T]^2$

Q.6 (A,B,D)

Mass = $M^0L^0T^0$

MVr = $M^0L^0T^0$

$M^0\frac{L^1}{T^1}\,.\,L^1 = M^0L^0T^0$

$L^2 = T^1$ ....(1)

Force = $M^1L^1T^{-2}$ (in SI)

= $M^0L^1L^{-4}$ (In new system from equation (1))

= L^{-3}

Energy = $M^1L^2T^{-2}$ (In SI)

= $M^0L^2L^{-4}$ (In new system from equation (1))

= L^{-2}

Power = $\frac{\text{Energy}}{\text{Time}}$

= $M^1L^2T^{-3}$ (in SI)

= $M^0L^2L^{-6}$ (In new system from equation (1))

= L^{-4}

Linear momentum = $M^1L^1T^{-1}$ (in SI)

= $M^0L^1L^{-2}$ (In new system from equation(1))

= L^{-1}

Q.7 (A, B)

Given $L=x^{\alpha}$ ....(1)

$LT^{-1}=x^{\beta}$ ....(2)

$LT^{-2}=x^{p}$ ....(3)

$MLT^{-1}=x^{q}$ ....(4)

$MLT^{-2}=x^{r}$ ....(5)

$\frac{(1)}{(2)} \Rightarrow T = x^{\alpha-\beta}$

From (3)

$$\frac{x^{\alpha}}{x^{2(\alpha-\beta)}} = x^{p}$$

$\Rightarrow \alpha + p = 2\beta$ (A)

From (4)

$M=x^{q-\beta}$

From (5) $x^q = x^r \; x^{\alpha-\beta}$

$\alpha+r-q=\beta$ .....(6)

Replacing value ‘α’ in equation (6) from (A)

$2\beta-p+r-q=\beta$

$\Rightarrow p+q-r=\beta$ (B)

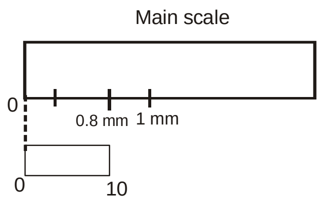
Replacing value of ‘β’ in equation (6) from (A)

$2\alpha+2r-2q=\alpha+p$

$\alpha=p+2q-2r$

Q.8 (BD)

Q.9 (D)



20 VSD = 16 MCD

1 VSD = 0.8 MSD

Least count MSD – VSD

= 1 mm – 0.8 mm

= 0.2 mm

Q.10 (B)

$\ell_1 = 52 + 1 = 53$ cm

$\ell_2 = 48 + 2 = 50$ cm

$$\frac{\ell_1}{\ell_2} = \frac{x}{R} \Rightarrow \frac{53}{50} = \frac{x}{10}$$

$x = 10.6\ \Omega$

Q.11 (C)

Least count = $\frac{0.5}{50}$ = 0.01 mm

Diameter of ball D = 2.5 mm + (20)(0.01)

D = 2.7 mm

$$\rho = \frac{M}{vol} = \frac{M}{\frac{4}{3}\pi\left(\frac{D}{2}\right)^3}$$

$$\left(\frac{\Delta\rho}{\rho}\right)_{max} = \frac{\Delta m}{m} + 3\frac{\Delta D}{D} \;;\; \left(\frac{\Delta\rho}{\rho}\right)_{max}$$

$$= 2\% + 3\left(\frac{0.01}{2.7}\right)\times 100\%$$

$$\frac{\Delta\rho}{\rho} = 3.1\%$$

Q.12 (A)

$\Delta d = \Delta\ell = \frac{0.5}{100}$ mm

$$y = \frac{4MLg}{\pi\ell d^2}$$

$$\left(\frac{\Delta y}{y}\right)_{max} = \frac{\Delta\ell}{\ell} + 2\frac{\Delta d}{d}$$

error due to ℓ measurement $\frac{\Delta\ell}{\ell} = \frac{0.5/100\,mm}{0.25\ mm}$

error due to d measurement

$$2\frac{\Delta d}{d} = \frac{2\times\frac{0.5}{100}}{0.5\ mm} = \frac{0.5/100}{0.25}$$

So error in y due to ℓ measurement = error in y due to d measurement

Q.13 (B)

50 VSD = 2.45 cm

1 VSD = $\frac{2.45}{50}$ cm = 0.049 cm

Least count of vernier = 1MSD – 1 VSD

= 0.05 cm – 0.049 cm

= 0.001 cm

Thickness of the object = Main scale reading + vernier scale reading × least count

= 5.10 + (24) (0.001)

= 5.124 cm.

Q.14 (A,B,D)

$$T = 2\pi\sqrt{\frac{7}{5}\frac{(R-r)}{g}}$$

$$\ln T = \ln 2\pi\sqrt{\frac{7}{5}} + \frac{1}{2}\ln(R-r) + \frac{1}{2}\ln g$$

$$\frac{\Delta g}{g} = 2\frac{\Delta T}{T} + \frac{\Delta R + \Delta r}{R-r}$$

$$\frac{\Delta g}{g}\times 100 = \left(2\times\frac{0.02}{0.556} + \frac{1+1}{60-10}\right)\times 100$$

$= 11.2\% \simeq 11\%$

$$\frac{\Delta r}{r}\times 100 = \frac{1}{10}\times 100 = 10\%$$

$$\frac{\Delta T}{T}\times 100 = \frac{0.02}{0.556}\times 100 = 3.57\%$$

Hence, (a, b, d)

Q.15 (D)

$2d \sin\theta = \lambda$

$d = \frac{\lambda}{2\sin\theta}$

differntiate

$\partial (d) = \frac{\lambda}{2} \partial (\text{cosec}\theta)$

$\partial (d) = \frac{\lambda}{2}(-\text{cosec}\theta \cot\theta)\partial\theta$

$\partial (d) = \frac{-\lambda\cos\theta}{2\sin^2\theta}\partial\theta$

as θ = increases , $\frac{\lambda\cos\theta}{2\sin^2\theta}$ decreases

2nd Method

$d = \frac{\lambda}{2\sin\theta}$

$\ell n\, d = \ell n\, \lambda - \ell n 2 - \ell n \sin\theta$

$\frac{\Delta(d)}{d} = 0 - 0 - \frac{1}{\sin\theta} \times \cos\theta(\Delta\theta)$

Fractional error $|+(d)| = |\cot\theta\, \Delta\theta|$

Absoulute error $\Delta d = (d\cot\theta)\, \Delta\theta$

$\frac{d}{2\sin\theta} \times \frac{\cos\theta}{\sin\theta}$

$\Delta d = \frac{\cos\theta}{\sin^2\theta}$

Q.16 [8]

Observation – 1

Let weight used is W_1, extension ℓ_1

$y = \frac{W_1/A}{\ell_1/L} \Rightarrow W_1 = \frac{yA\ell_1}{L}$

$\ell_1 = 3.2\times10^{-2} + 20\times10^{-5}$

Observation – 2

Let weight used is W_2 extension ℓ_2

$y = \frac{W_2/A}{\ell_2/L} \Rightarrow W_1 = \frac{yA\ell_2}{L}$

$\ell_1 = 3.2\times10^{-2} + 45\times10^{-5}$

$W_2 - W_1 = \frac{yA}{L}(\ell_2 - \ell_1) \Rightarrow y = \frac{(W_2 - W_1)/L}{yA(\ell_2 - \ell_1)}$

$\left(\frac{\Delta y}{y}\right)_{max} = \frac{\Delta\ell_2 + \Delta\ell_1}{\ell_2 - \ell_1} = \frac{2\times10^{-5}}{25\times10^{-5}}$

$\left(\frac{\Delta y}{y}\right)_{max} \times 100\% = \frac{2}{25} \times 100\% = 8\%$

Q.17 (D)

$R_1 = 2.8 + 0.01 \times 7 = 2.87$

$R_2 = 2.8 + (8MSD - 7VSD) = 2.8 + (8\times 0.1 - 7 \times \frac{11}{10}) = 2.83$

Q.18 (D)

$t = \sqrt{\frac{L}{5}} + \frac{L}{300}$

$dt = t = \frac{1}{\sqrt{5}}\frac{1}{2}L^{-1/2}dL + \left(\frac{1}{300}dL\right)$

$dt = \frac{1}{2\sqrt{5}}\frac{1}{\sqrt{20}}dL + \frac{dL}{300} = 0.01$

$dL\left(\frac{1}{20} + \frac{1}{300}\right) = 0.01$

$dL\left[\frac{15}{300}\right] = 0.01$

$dL = \frac{3}{16}$

$\frac{dL}{L} \times 100 = \frac{3}{16} \times \frac{1}{20} \times 100 = \frac{15}{16} \simeq 1\%$

Q.19 (B)

$r = \frac{(1-a)}{(1+a)}$

$\frac{\Delta(1-a)}{(1-a)} + \frac{\Delta(1+a)}{(1+a)}$

$= \frac{\Delta a}{(1-a)} + \frac{\Delta a}{(1+a)}$

$= \frac{\Delta a(1+a+1-a)}{(1-a)(1+a)}$

$\therefore \Delta r = \frac{2\Delta a}{(1-a)(1+a)}\frac{(1-a)}{(1+a)} = \frac{2\Delta a}{(1+a)^2}$

Q.20 (C)

$N = N_0 e^{-\lambda t}$

$\ln N = \ln N_0 - \lambda t$

$$\frac{dN}{N} = -d\lambda t$$

Converting to erros.

$$\frac{\Delta N}{N} = \Delta\lambda t$$

$$\therefore\ \Delta\lambda = \frac{40}{2000 \times L} = 0.02$$

(N is number of nuclei left undecayed.)

Q.21 [3.00]

d = 0.5 mm Y = 2 × 10^{11} ℓ = 1m

$$\Delta\ell = \frac{F\ell}{Ay} = \frac{mg\ell}{\frac{\pi d^2}{4} y} = \frac{1.2 \times 10 \times 1}{\frac{\pi}{4} \times \left(5 \times 10^{-4}\right)^2 \times 2 \times 10^{11}}$$

$$\Delta\ell = \frac{1.2 \times 10}{\frac{3.2}{4} \times 25 \times 10^{-8} \times 2 \times 10^{11}}$$

$$= \frac{12}{0.8 \times 25 \times 2 \times 10^3} = \frac{12}{40 \times 10^3} = 0.3\text{mm}$$

so 3^{rd} division of vernier scale will coincicle with main scale.

Q.22 [1.30]

$$U = \frac{1}{2}\frac{C_1 C_2}{C_1 + C_2} V^2$$

Let $C_{eq} = \dfrac{C_1 C_2}{C_1 + C_2}$

$$\frac{1}{C_{eq} \pm \Delta C_{eq}} = \frac{1}{C_1 \pm \Delta C_1} + \frac{1}{C_2 \pm \Delta C_2}$$

$$\Rightarrow C_{eq} \pm \Delta C_{eq} \simeq \frac{C_1 C_2 + C_1 \Delta C_2 + C_2 \Delta C_1}{C_1 + C_2 + \Delta C_1 + \Delta C_2}$$

$$= \frac{1200\left(1 \pm \frac{12}{1200}\right)}{\left(1 \pm \frac{25}{5000}\right)}$$

$$= 1200\left[1 \pm \left(\frac{1}{100} - \frac{1}{200}\right)\right]$$

$$\frac{\Delta U}{U} \times 100 = \frac{\Delta C_{eq}}{C_{eq}} \times 100 + \frac{2\Delta V}{V} \times 100$$

$$= \frac{1}{200} \times 100 + 2 \times \frac{0.02}{5} \times 100$$

=1.3%

Q.23 (C)

Given 10 VSD = 9 MSD

Here MSD $\rightarrow$ Main scale division

1 VSD = $\dfrac{9}{10}$ MSD

VSD $\rightarrow$ Vernier Scale division

Least count = 1 MSD – 1 VSD

$$= \left(1 - \frac{9}{10}\right) \text{MSD}$$

= 0.1 MSD

= 0.1 × 0.1 cm

= 0.01 cm

As '0' of V.S. lie before '0' of M.S.

Zero error = –[10 – 6]L.C.

= – 4 × 0.01 cm

= – 0.04 cm

Reading = 3.1 cm + 1 × LC

= 3.4 cm + 1 × 0.01 cm

= 3.11 cm

True diameter = Reading – Zero error

= 3.11 – (–0.04) cm = 3.15 cm

Vector and Calculus

EXERCISES

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 (2)

Q.2 (2)
$|P-Q| \le R \le |P+Q|$
If $P = 10N$ & $Q = 6N$
$4 \le R \le 16$

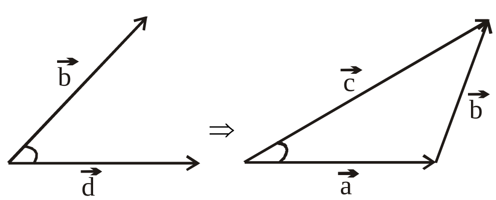
Q.3 (4)
Initial & final position are coincide.

Q.4 (4)
$R_{min} = |P-Q|$
When angle between P & Q is 180º.

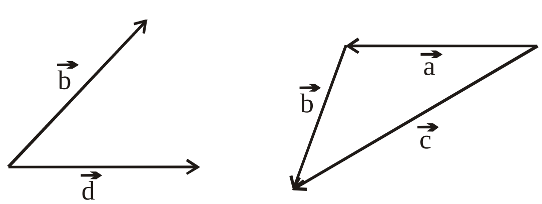
Q.5 (1)
$R_{max} = (P+Q)$
When angle between P & Q is 0º

Q.6 (3)
$R^2 = P^2 + Q^2 + 2PQ\cos\theta$
$R = 13$
$P = 5$
$Q = 12$
$\cos\theta = 0$
$\theta = \frac{\theta}{2}$

Q.7 (1)



Now direction one interchanged



Change in direction but angle between A & B is change so no change in magnitude.

Q.8 (4)
$\vec{P} + \vec{Q} = \vec{P} - \vec{Q}$
$\overrightarrow{2Q} = 0$
$\vec{Q} = 0$
$|\vec{Q}| = 0$

Q.9 (4)

Q.10 (3)
$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$
$= 2\hat{i} - 3\hat{j} + 4\hat{k} - \hat{i} - \hat{j} + \hat{k}$
$= \hat{i} - 4\hat{j} + 5\hat{k}$

Q.11 (1)
$\boxed{2\hat{i} - 3\hat{j}}$ $\boxed{\vec{F} = F_x\hat{i} + F_y\hat{j}}$

Q.12 (4)
$\vec{A}.\vec{B} = 0$ – $A \perp B$
$\vec{A}.\vec{C} = 0$ – $A \perp C$
If may be possible
$A \perp B \perp C$
Than A must be perpendicular to $\vec{B} \times \vec{C}$.

Q.13 (2)
$AB\cos\theta = 8$
$AB\sin\theta = 8\sqrt{3}$
$\tan\theta = \sqrt{3}$
$\theta = 60º$

JEE-ADVANCED
OBJECTIVE QUESTIONS

Q.1 (1)
θ will increase plan cos θ will decreases.

Q.2 (2)
Net displacement must be zero.

Q.3 (2)

$|P - Q| \le R \le (P + Q)$

Q.4 (4)

$|\overline{A} + \overline{B}| = \overline{A} = \overline{B}$

$A^2 + B^2 + 2AB\cos\theta = A^2$

$2A^2 + 2A^2\cos\theta^o = A^2$

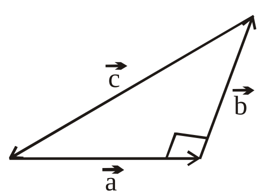
$2A^2 + 2A^2\cos\theta^o = A^2$

$2\cos\theta = -1$

$\cos\theta = -\frac{1}{2}$

$\theta = 120^o$

Q.5 (1)



$\overline{a} + \overline{b} = -\overline{c}$

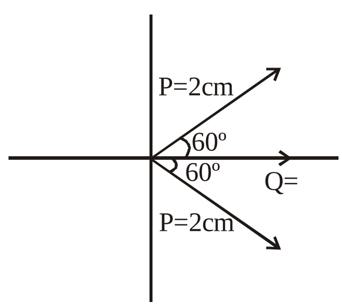
$c^2 = a^2 + b^2 + 2ab\cos\theta$

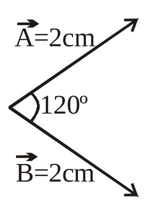
$\overline{a} = \overline{b}$

$c = \sqrt{2}a = \sqrt{2}b$

$\boxed{\theta = 90^o}$

Q.6 (2)





Resulting of $\overline{A} \,\&\, \overline{B}$ also have same unit 2 cm.

Q.7 (1)

By polygon law for vector addition.

Q.8 (3)

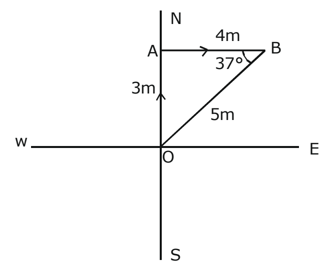
$R^2 = P^2 + Q^2 + 2PQ\cos 60°$

$7Q^2 = P^2 + Q^2 + PQ \Rightarrow P^2 - 6Q^2 + PQ = 0$

$$\left(\frac{P}{Q}\right)^2 + \left(\frac{Q}{Q}\right) - 6 = 0$$

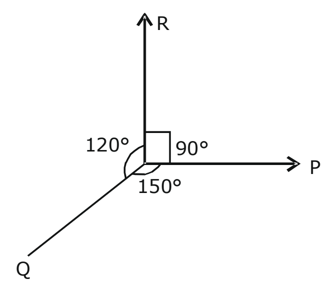
$$\Rightarrow \left(\frac{P}{Q} + 3\right)\left(\frac{P}{Q} - 2\right) = 0 \Rightarrow \frac{P}{Q} = 2$$

Q.9 (2)



Displacement = 0

Q.10 (4)



$$\frac{P}{\sin 120°} = \frac{Q}{\sin 90°} = \frac{R}{\sin 150°}$$

$$\frac{P}{\sqrt{3/2}} = \frac{\theta}{1} = \frac{R}{\frac{1}{2}}$$

$P : Q\ R = \sqrt{3} : 2 : 1$

Q.11 (4)

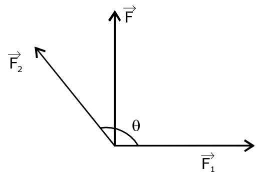
$\vec{A} = \hat{i} - 2\hat{j} + 3\hat{k}$

$\vec{B} = 4\hat{i} + 2\hat{j} + 3\hat{k} \Rightarrow \overrightarrow{AB} = 3\hat{i} + 4\hat{j}$

$$\hat{AB} = \frac{3\hat{i} + 4\hat{j}}{5} \Rightarrow \vec{V} = 10\left(\frac{3\hat{i} + 4\hat{j}}{5}\right) = 6\hat{i} + 8\hat{j}$$

Q.12 (4)

$|\vec{F_2}| = |\vec{F_2}|$



$$\tan\alpha = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta} \Rightarrow \alpha = 90°$$

then $F_1 + F_2\cos\theta = 0 \Rightarrow \cos\theta = -\frac{1}{2}$

$\theta = 120°$

Q.13 (1)

$|\vec{F_1}| = |\vec{F_2}| = \text{dyne} \Rightarrow \theta = 120°$

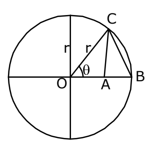
$|\vec{F}| = 2|\vec{F_1}|\cos\theta/2 \Rightarrow 10 \text{ dyne}$

Q.14 (1)

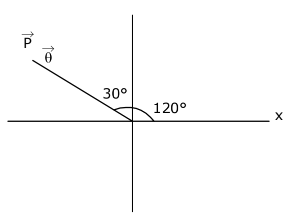
Displacement $BC = \sqrt{AB^2 + AC^2}$

$\sqrt{(r - r\cos\theta)^2 + r^2\sin^2\theta}$

$BC = 2r\sin\theta/2$



Q.15 (1)

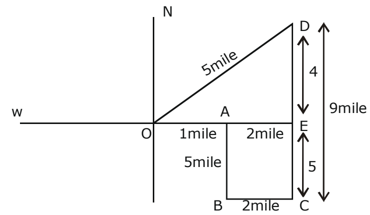


Angle b/w $\vec{P}$ & $\vec{\theta}$ is 0

so Resultant $= \sqrt{P^2 + \theta^2 + 2PQ\cos\theta}$

$|\vec{R}| = P + Q$

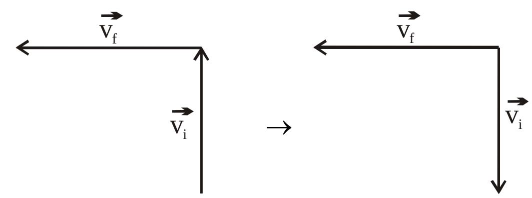
Q.16 (2)

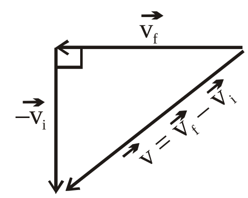


$OD^2 = OE^2 + ED^2 \Rightarrow OD = \sqrt{3^2 + 4^2} = 5\text{miles}$

Q.17 (2)

$\overrightarrow{\Delta_v} = \vec{v_f} - \vec{v_i}$

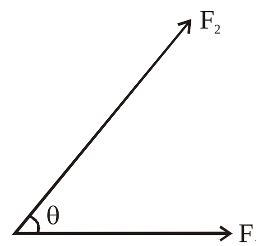




$|v_f - v_i| = 50\sqrt{2}$

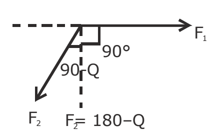
Q.18 (1)

$P^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$



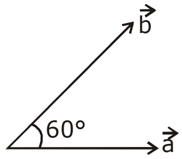
$Q^2 = F_1^2 + F_2^2 - 2F_1F_2\cos\theta$

$P^2 + Q^2 = 2(F_1^2 + F_2^2)$



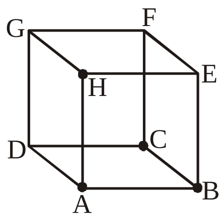
Q.19 (2)

$|\hat{a} - \hat{b}| = \sqrt{a^2 + b^2 - 2ab\cos\theta}$



$\theta = 60° \Rightarrow |\hat{a} - \hat{b}| = 1$

Q.20 (4)



Start from A to F

$AC^2 = AB^2 + CB^2$

$AC^2 = 10^2 + 12^2$

$AC^2 = 100 + 144 = 244$

$AF^2 = AC^2 + CF^2 = 244 + 196$

$AF^2 = 440$

$AF = 20.99 \simeq 21$

Q.21 (3)

$\bar{A} + \bar{B} = 7\hat{i} - 3\hat{j}$

$|A + B| = \sqrt{4a + 9} = \sqrt{58}$

Q.22 (1)

$\bar{A} - \bar{B} = 3\hat{i} - \hat{k}$

$A - B = \dfrac{\bar{A} - \bar{B}}{|A - B|} = \dfrac{3\hat{i} - \hat{k}}{\sqrt{10}}$

Q.23 (2)

$\vec{B} = x\vec{a}$

on multiplying with the scalar magnitude will change

if x is –ve direction of $\vec{B}$ change

if x is +ve direction of $\vec{B}$ same as $\vec{a}$

$\vec{B}$ & $\vec{a}$ are colinear vector

Q.24 (2)

$\vec{A} = 2\hat{i} + 3\hat{j}$

$\vec{B} = \hat{j} \Rightarrow \vec{A}.\vec{B} = |\vec{A}||\vec{B}|\cos\theta$

$\cos\theta = \dfrac{3}{\sqrt{13}} \Rightarrow \tan\theta = 2/3 \Rightarrow \theta = \tan^{-1}(2/3)$

Q.25 (2)

$\vec{A} = 3\hat{i} + 2\hat{j} + 8\hat{k}$

$\vec{B} = 2\hat{i} + x\hat{j} + \hat{k}$

$\vec{A}.\vec{B} = |\vec{A}||\vec{B}|\cos\theta \quad \theta = 90°$

$\vec{A}.\vec{B} = 0$

$6 + 2x + 8 = 0, x = -7$

Q.26 (1)

$\vec{A} = a_1\hat{i} + a_2\hat{j}$

$\vec{B} = 4\hat{i} - 3\hat{j}$

$|\hat{A}| = 1 \Rightarrow a_1^2 + a_2^2 = 1$

...(i)

$\vec{A}.\vec{B} = 0$

$4a_1 - 3a_2 = 0$

$4a_1 = 3a_2$

...(ii)

(i) (ii) $\rightarrow a_1^2 + \dfrac{16}{9}a_1^2 = 1$

$a_1^2 = \dfrac{9}{15}, \; a_1 = \dfrac{3}{5} = 0.6, \quad a_2 = 0.8$

Q.27 (1)

By the defination of equal vector.

Q.28 (4)

$(\bar{A} \times \bar{B}) \perp A$

$(\bar{A} \times \bar{B}) \perp B$

JEE-ADVANCED

MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 (A,B,D)

Q.2 (A,B,C)

By right handed co-ordinate system.

Q.3 (C)

(Easy) $x = t^3 - 3t^2 + 12t + 20$

$v = \dfrac{dx}{dt} = 3t^2 - 6t + 12$

$t = 0 \Rightarrow v = 12$ m/s

Q.4 (D)

(Easy) $a = \dfrac{dv}{dt} = 6t - 6$

$t = 0 \Rightarrow a = -6$ m/s^2

Q.5 (D)

(Moderate) $a = 0 \Rightarrow t = 1$ sec.

$v = 3t^2 - 6t + 12 = 9$ m/s

Q.6 (A)

$\vec{F}_R = \vec{F}_1 + \vec{F}_2 = 2\hat{i} + 5\hat{j} + 4\hat{k}$

Q.7 (B)

$\cos\theta = \dfrac{\vec{F}_1 \cdot \vec{F}_2}{|\vec{F}_1||\vec{F}_2|} \Rightarrow \theta = \cos^{-1}\left(\dfrac{3}{5\sqrt{2}}\right)$

Q.8 (C)

$F_1 \cos\theta = \dfrac{\vec{F}_1 \cdot \vec{F}_2}{|\vec{F}_2|} = \dfrac{6}{5}$

Q.9 (C)

Q.10 (A)

Q.11 (A)

Q.12 (A) $\to$ r, (B) $\to$ p, (C) $\to$ q, (D) $\to$ s

(A) $\int \sec x \tan x \, dx = \sec x + C$

(B) $\int \text{cosec}\, kx \cot kx \, dx = \dfrac{\cos kx}{k} + C$

(C) $\int \text{cosec}^2 kx \, dx = -\dfrac{\cot kx}{k} + C$

(D) $\int \cos dx = \dfrac{\sin kx}{k} + C$

Q.13 (A) $\to$ q, (B) $\to$ r, (C) $\to$ p, (D) $\to$ s

(A) $|\vec{A} + \vec{B}| = A^2 + B^2 + 2AB\cos\theta$

$A^2 = A^2 + A^2 + 2A^2\cos\theta \; \cos\theta = -\dfrac{1}{2} \; \theta = 120$

(B) $F_1 \sim F_2 \le R \le F_1 + F_2$

Here $F_1 \sim F_2 = 4$ and $F_1 + F_2 = 12$

(C) $\cos\theta = \dfrac{\vec{A} - \vec{B}}{|A| + |B|} = \dfrac{0}{2\sqrt{2} \times 3} = 0 \; \theta = 90°$

(D) $\vec{A} + \vec{B} = 2\hat{i} + \hat{j} + 3\hat{k}$

$|\vec{A} + \vec{B}| = \sqrt{2^2 + 1 + 3^2} = \sqrt{14}$

NUMERICAL VALUE BASED

Q.1 [1]

(Given |A| = 3)

$|\vec{A}| - |\vec{B}| = 2$

$|\vec{B}| > |\vec{A}|$

$|\vec{B}| = 1$

Q.2 [1]

$|\vec{A}| + |\vec{B}| = 5$

$2 + |\vec{B}| = 5$

$|\vec{B}| = 3$

resultant of $|\vec{A}|$ and $|\vec{B}|$

$|\vec{B}| - |\vec{A}| = 3 - 2 = 1$

Q.3 [2]

$\vec{v}_1 . \vec{v}_2 = 0 \Rightarrow t = 2$ sec

Q.4 [1]

$\vec{a} = \dfrac{d^2\vec{r}}{dt^2}$

$\vec{r}$ and $\vec{a}$ are perpendicular if $\vec{r} \cdot \vec{a} = 0$

Q.5 [1]

(Given |A| = 3)

$|\vec{A}| - |\vec{B}| = 2 \qquad |\vec{B}| > |\vec{A}|$

$|\vec{B}| = 1$

Q.6 [1]

$|\vec{A}| + |\vec{B}| = 5$

$2 + |\vec{B}| = 5$

resultant of $|\vec{A}|$ and $|\vec{B}|$

$|\vec{B}| - |\vec{A}| = 3 - 2 = 1$

KVPY
PREVIOUS YEAR'S

Q.1 (D)

$(\hat{n} \times \vec{F}) \times \hat{n} = -[\hat{n} \times (\hat{n}.\vec{F})]$

$= -[\hat{n}(\hat{n}.\vec{F}) - \vec{F}(\hat{n}.\hat{n})]$

$= \vec{F} - \hat{n}(\hat{n}.\vec{F})$

$= \vec{G}$

JEE-MAIN PREVIOUS YEAR'S

Q.1 [8]

$$\frac{\vec{a}+\vec{b}+\vec{c}+\vec{d}+\vec{e}+\vec{f}+\vec{g}+\vec{h}}{8}=0$$

$$\vec{b}+\vec{c}+\vec{d}+\vec{e}+\vec{f}+\vec{g}+\vec{h}=-\vec{a}$$

$$\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}+\overrightarrow{AG}+\overrightarrow{AH}$$

$$\vec{b}-\vec{a}+\vec{c}-\vec{a}+\vec{d}-\vec{a}+\vec{e}-\vec{a}+\vec{f}-\vec{a}+\vec{g}-\vec{a}+$$

$$\vec{h}-\vec{a}$$

$$\left(\vec{b}+\vec{c}+\vec{d}+\vec{e}+\vec{f}+\vec{g}+\vec{h}\right)-7\vec{a}$$

$$\Rightarrow\ -\vec{a}-7\vec{a}$$

$$= -8\vec{a}=-8(\overrightarrow{OA})=8\overrightarrow{AO}=8(\overrightarrow{AO})$$

Q.2 [180°]

$\vec{p}\times\vec{g}=\vec{g}\times\vec{p}$ only if $\vec{p}=0$ or $\vec{g}=0$ or angle between them is 0° or 180°.

$\therefore\ \theta=180^\circ$

Q.3 (4)
Q.4 (4)
Q.5 [3]
Q.6 (4)
Q.7 (2)
Q.8 (2)
Q.9 (4)
Q.10 (3)
Q.11 (2)

$$\vec{A}=\vec{P}+\vec{Q}$$

$$\vec{B}=\vec{P}-\vec{Q} \qquad \vec{P}\perp\vec{Q}$$

$$|\vec{A}|=|\vec{B}|=\sqrt{P^2+Q^2}$$

$$|\vec{A}+\vec{B}|=\sqrt{2(P^2+Q^2)(1+\cos\theta)}$$

For $|\vec{A}+\vec{B}|=\sqrt{3(P^2+Q^2)}$

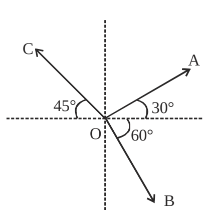
$\theta_1=60^\circ$

For $|\vec{A}+\vec{B}|=\sqrt{2(P^2+Q^2)}$

$\theta_2=90^\circ$

Q.12 (1)

Q.13 (1)



Let magnitude be equal to λ.

$$\overrightarrow{OA}=\lambda\left[\cos30^\circ\hat{i}+\sin30\hat{j}\right]=\lambda\left[\frac{\sqrt{3}}{2}\hat{i}+\frac{1}{2}\hat{j}\right]$$

$$\overrightarrow{OB}=\lambda\left[\cos60^\circ\hat{i}-\sin60\hat{j}\right]=\lambda\left[\frac{1}{2}\hat{i}-\frac{\sqrt{3}}{2}\hat{j}\right]$$

$$\overrightarrow{OC}=\lambda\left[\cos45^\circ\left(-\hat{i}\right)+\sin45\hat{j}\right]=\lambda\left[-\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{j}\right]$$

$$\therefore\ \overrightarrow{OA}+\overrightarrow{OB}-\overrightarrow{OC}$$

$$=\lambda\left[\left(\frac{\sqrt{3}+1}{2}+\frac{1}{\sqrt{2}}\right)\hat{i}+\left(\frac{1}{2}-\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}\right)\hat{j}\right]$$

$\therefore$ Angle with x-axis

$$\tan^{-1}\left[\frac{\frac{1}{2}-\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2}+\frac{1}{2}+\frac{1}{\sqrt{2}}}\right]=\tan^{-1}\left[\frac{\sqrt{2}-\sqrt{6}-2}{\sqrt{6}+\sqrt{2}+2}\right]$$

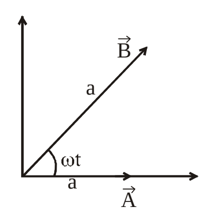
$$=\tan^{-1}=\left[\frac{1-\sqrt{3}-\sqrt{2}}{\sqrt{3}+1+\sqrt{2}}\right]$$

JEE-ADVANCED PREVIOUS YEAR'S

Q.1 (D)

$$\vec{S}=\vec{P}+b\vec{R}=\vec{P}+b\left(\vec{Q}-\vec{P}\right)=\vec{P}\left(1-b\right)+b\vec{Q}$$

Q.2 [2.00 sec]



$$|\vec{A}+\vec{B}|=2a\cos\frac{\omega t}{2}$$

$$|\vec{A}+\vec{B}|=2a\sin\frac{\omega t}{2}$$

$$\text{So, } 2a\cos\frac{\omega t}{2}=\sqrt{3}\left(2a\sin\frac{\omega t}{2}\right)$$

$$\tan\frac{\omega t}{2}=\frac{\pi}{6}\Rightarrow\omega t=\frac{\pi}{3}$$

$$\frac{\pi}{6}t=\frac{\pi}{3}$$

t = 2.00 sec

EXERCISES

ELEMENTARY

Q.1 (1)

$\vec{r} = 20\hat{i} + 10\hat{j}$

$\therefore r = \sqrt{20^2 + 10^2} = 22.5m$

Q.2 (2)
Total time of motion is 2 min 20 sec = 140 sec.
As time period of circular motion is 40 sec so in 140 sec. athlete will complete 3.5 revolution i.e., He will be at diametrically opposite point i.e., Displacement = 2R.

Q.3 (4)
As the total distance is divided into two equal parts therefore distance averaged speed $= \frac{2v_1v_2}{v_1 + v_2}$.

Q.4 (3)
Total distance to be covered for crossing the bridge
= length of train + length of bridge
$= 150m + 850m = 1000m$

$\text{Time} = \frac{\text{Distance}}{\text{Velocity}} = \frac{1000}{45 \times \frac{5}{18}} = 80\,\text{sec}.$

Q.5 (3)
From given figure, it is clear that the net displacement is zero. So average velocity will be zero.

Q.6 (2)

As $S = ut + \frac{1}{2}at^2 \; \therefore S_1 = \frac{1}{2}a(10)^2 = 50a$ .....(i)

As $v = u + at$

velocity acquired by particle in 10 sec $v = a \times 10$

For next 10 sec, $S_2 = (10a) \times 10 + \frac{1}{2}(a) \times (10)^2$

$S_2 = 150a$ .....(ii)
From (i) and (ii) $S_1 = S_2/3$

Q.7 (4)

$\vec{a} = \frac{\vec{F}}{m}$. . If $\vec{F} = 0$ then $\vec{a} = 0$.

Q.8 (2)
From $v^2 = u^2 + 2aS$
$\Rightarrow 0 = u^2 + 2aS$

$\Rightarrow a = \frac{-u^2}{2S} = \frac{-(20)^2}{2 \times 10} = -20m/s^2.$

Q.9 (2)
Constant velocity means constant speed as well as same direction throughout.

Q.10 (4)
$S \propto u^2$ If u becomes 3 times then S will become 9 times i.e. $9 \times 20 = 180m$.

Q.11 (4)
$x \propto t^3 \; \therefore x = Kt^3$

$\Rightarrow v = \frac{dx}{dt} = 3Kt^2$ and $a = \frac{dv}{dt} = 6Kt$

i.e. $\alpha \propto t$..

Q.12 (3)
$u = 12$ m/s, $g = 9.8$ m/sec^2, $t = 10$ sec

Displacement $= ut + \frac{1}{2}gt^2$

$= 12 \times 10 + \frac{1}{2} \times 9.8 \times 100 = 610m.$

Q.13 (4)
A body A is projected upwards with a velocity of 98 m/s. The second body B is projected upwards with the same initial (4) Let t be the time of flight of the first body after meeting, then $(t-4)$ sec will be the time of flight of the second body. Since $h_1 = h_2$

$\therefore 98t - \frac{1}{2}gt^2 = 98(t-4) - \frac{1}{2}g(t-4)^2$

On solving, we get t = 12 seconds

Q.14 (3)
Vertical component of velocities of both the balls are same and equal to zero. So $t = \sqrt{\frac{2h}{g}}$.

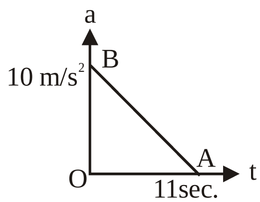
Q.15 (1)

$h = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times (4)^2 = 80m.$

Q.16 (2)

$t = \sqrt{\frac{2h}{g}} \Rightarrow \frac{t_1}{t_2} = \sqrt{\frac{h_1}{h_2}}.$

Q.17 (2)
The area under acceleration time graph gives change in velocity. As acceleration is zero at the end of 11 sec

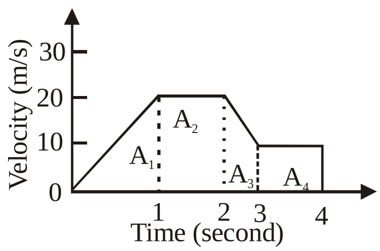


i.e. v_{max} Area of ΔOAB

$$= \frac{1}{2} \times 11 \times 10 = 55\,m/s$$

Q.18 (2)

Distance = Area under v — t graph = $A_1 + A_2 + A_3 + A_4$



$$= \frac{1}{2} \times 1 \times 20 + (20 \times 1) + \frac{1}{2}(20+10) \times 1 + (10 \times 1)$$

$= 10 + 20 + 15 + 10 = 55\,m.$

Q.19 (2)

Region OA shows that graph bending toward time axis i.e. acceleration is negative.

Q.20 (3)

Area of trapezium $= \frac{1}{2} \times 3.6 \times (12+8) = 36.0\ m$.

Q.21 (4)

Slope of displacement time graph is negative only at point E.

Q.22 (2)

Maximum acceleration will be represented by CD part of the graph

$$\text{Acceleration} = \frac{dv}{dt} = \frac{(60-20)}{0.25} = 160\ km/h^2$$

Q.23 (3)

From given a – t graph it is clear that acceleration is increasing at constant rate

$\therefore \frac{da}{dt} = k$ (constant) $\Rightarrow$ a = kt (by integration)

$$\Rightarrow \frac{dv}{dt} = kt \Rightarrow dv = ktdt$$

$$\Rightarrow \int dv = k\int tdt \Rightarrow v = \frac{kt^2}{2}$$

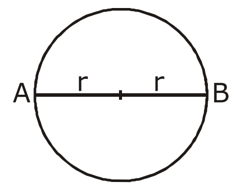
i.e. v is dependent on time parabolically and parabola is symmetric about v-axis.

and suddenly acceleration becomes zero. i.e. velocity becomes constant.

Hence (3) is most probable graph.

JEE-MAIN
OBJECTIVE QUESTIONS

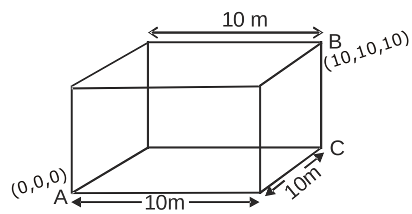
Q.1 (2)



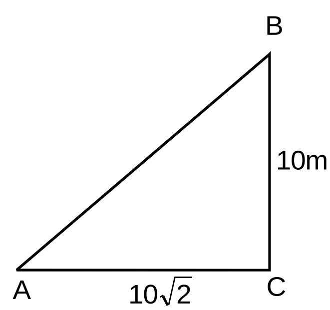
Displacement = 2r

distance = πr

Q.2 (2)



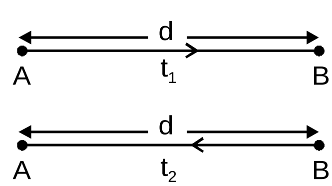
Fly start from A and reaches at B.



$$\therefore \quad AB = \sqrt{(10\sqrt{2})^2 + 10^2} = 10\sqrt{3}m$$

Q.3 (2)

From A to B $t_1 = \frac{d}{20}$ hr $\Rightarrow$ From B to A $t_2 = \frac{d}{30}$ hr



$\therefore$ Average Speed $= \dfrac{\text{Total Distance}}{\text{Total Time}}$

$= \dfrac{2d}{t_1 + t_2} = \dfrac{2d}{\dfrac{d}{20} + \dfrac{d}{30}} \Rightarrow \quad v = 24 \text{ km/hr}$

Q.4 (2)

Average velocity $= \dfrac{\text{Total Distance}}{\text{Total time}}$

$48 = \dfrac{2000}{\dfrac{1000}{40} + \dfrac{1000}{V}} = \dfrac{80V}{40 + V} \Rightarrow V = 60$

Q.5 (2)

$V_x = 2at$

$V_y = 2bt$

$V = 2t\sqrt{a^2 + b^2}$

Q.6 (2)

$t = 62.8$ sec

in each lap car travel a distance $= 2\pi R$

$= 2 \times 3.14 \times 100 = 628$ m

In each lap displacement of the car = 0

Average speed

$= \dfrac{\text{Total Distance}}{\text{Total Time}} = \dfrac{628}{62.8} = 10 \text{ m/s}$

Average Velocity $= \dfrac{\text{Total Displacement}}{\text{Total Time}} = 0$

Q.7 (1)

$2s = gt^2 \Rightarrow s = \dfrac{1}{2}gt^2$

$v = \dfrac{ds}{dt} = gt$

Q.8 (3)

$V_{inst} = \dfrac{dx}{dt}$ (slop of x-t graph)

At C tan θ =+ve At E $\theta > 90^o(-\text{ve slop})$

At D $\theta = 0^o$ At F $\theta < 90^o(+\text{ve slope})$ $\therefore$ At E v_{inst} is negative

Q.9 (3)

let the acceleration of the body is a and $u = 0$

then $x_1 = \dfrac{1}{2}at^2 = \dfrac{1}{2}a(10)^2$

$x_2 = \dfrac{1}{2}a(20)^2 - x_1 \Rightarrow x_2 = \dfrac{1}{2}a(20)^2 - \dfrac{1}{2}a(10)^2$

$= \dfrac{1}{2}a(10)(30) \Rightarrow x_3 = \dfrac{1}{2}a(30)^2 - \dfrac{1}{2}a(20)^2$

$= \dfrac{1}{2}a(10)(50) \Rightarrow \therefore x_1 : x_2 : x_3 = 1:3:5$

Q.10 (1)

u = 10m/sec a = - 2m/sec^2

Total time taken when final is zero.

a = 10m/sec^2

10m/sec

v = 0

0 = 10-2t

t = 5 sec

$S = ut + \dfrac{1}{2}at^2$

$S_{t=5} = 10 \times 5 - \dfrac{1}{2} \times 2 \times 25 = 25$

$S_{t=4\text{ sec.}} = 10 \times 4 - \dfrac{1}{2} \times 2 \times 16 = 24$

$S_{t=5} - S_{t=4} = 25 - 24 = 1$ m

Q.11 (2)

$S_2 = \dfrac{1}{2} \times a \times 4$

$S_3 = \dfrac{1}{2} \times a \times 9$

$S_4 = \dfrac{1}{2} \times a \times 16$

$S_5 = \dfrac{1}{2} \times a \times 25$

distance travelled by body in 3rd see = $\dfrac{1}{2}a[7]$

distance travelled by body in 4th see = $\dfrac{1}{2}a[9]$

ratio = 7 : 9

Q.12 (4)

Let constant acceleration = a

$S = \dfrac{1}{2}at^2$

$$S_1 = \frac{1}{2}a \times 10^2 = 50a$$

$$S_2 = \frac{1}{2}a \times 20^2 - \frac{1}{2}a \times 10^2 = 150a$$

$S_2 = 3S_1$

Q.13 (2)
Total Length of 2 trains = 50 + 50 = 100
Velocity $V_1 = 10$
$V_2 = 15$
$V_1 + V_2 = 25$

$$\text{time} = \frac{100}{25} = 4\,\text{sec}$$

Q.14 (2)
In inclined initial u = 0

$$S = \frac{1}{2}at^2 \text{ and } a = g\sin\theta$$

$$l = \frac{1}{2}g\sin\theta\, x \times (4)^2$$

...(i)

$$\frac{\ell}{4} = \frac{1}{2}g\sin\theta\, t^2$$

...(ii)
From (i) and (ii)
t = 2 sec

Q.15 (3)

$$\frac{h}{2} = \frac{1}{2}gt_1^2$$

....(1)

$$h = \frac{1}{2}g(t_1 + t_2)^2 \quad ....(2)$$

From equation (1) and (2)

$$2t_1^2 = (t_1 + t_2)^2$$

$$\sqrt{2}t_1 = t_1 + t_2$$

$$(\sqrt{2} - 1)t_1 = t_2$$

$$t_1 = \frac{t_2}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$$

$$t_1 = \left(\sqrt{2}+1\right) t_2$$

Q.16 (3)
Acceleration for both is g

$$a = \frac{1}{2}gt_a^2$$

$$b = \frac{1}{2}t_b^2$$

$$t_a : t_b = \sqrt{a} : \sqrt{b}$$

Q.17 (4)
At H_{max}, v = 0
Acceleration constant & it is due to gravity

$$|a| = g$$

Q.18 (4)
Horizontal Component of velocity
Because there is no acceleration in horizontal Direction

Q.19 (4)

$$v = \frac{ds}{dt} = 3t^2 - 12t + 3$$

$$a = \frac{dv}{dt} = 6t - 12$$

$$a = 0 \Rightarrow t = 2\,\text{sec}$$

$$V_{2\,sec} = 3(2)^2 - 12(2) + 3 = +12 - 24 + 3$$

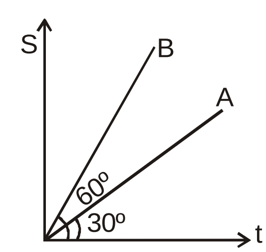
= – 9 m/s

Q.20 (3)
From graph it is clear that velocity is always positive during its motion
so displacement = distance
displacement = Area under V-t curve

$$= \frac{1}{2} \times 20 \times 1 + 20 \times 1 + \frac{1}{2} \times 1 \times 30 + 1 \times 10$$

$\Rightarrow$ 55 m

Q.21 **(4)**



$$\frac{V_A}{V_B} = \frac{\tan\ 30^o}{\tan\ 60^o} \Rightarrow \therefore\ \frac{V_A}{V_B} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3}$$

Q.22 (4)

Total Distance = Area

under the curve (Position + Negative)

$$= \frac{1}{2}\times 4\times 1 + 4\times 2 + 1\times 4\times \frac{1}{2} - \frac{1}{2}\times 2\times 1 - 2\times 2 - \frac{1}{2}\times 1\times 2$$

= 2 + 8 + 2 – 1 – 4 – 1 = 6 meter

Q.23 (1)

$$\text{(acceleration)} = \text{Slope} = \frac{\Delta v}{\Delta t}$$

$$OA \rightarrow \frac{10}{10} = 1$$

$$AB \rightarrow 0 = 0$$

$$BC \rightarrow \frac{-10}{20} = -0.5$$

Q.24 (2)

Distance = Total Area

$= 105\,m$

Displacement = 90 - 15 = 75m

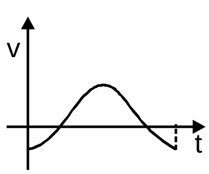
(-ve y asxis area) - (-ve y axis area)

Q.25 (3)

Equation of given sin curve is

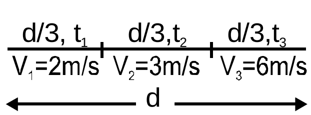
$$x = -A\sin t$$

$$V = \frac{dx}{dt} = -A\cos t$$



JEE-ADVANCED
OBJECTIVE QUESTIONS

Q.1 (A)



Now $t_1 = \dfrac{d/3}{v_1} = \dfrac{d}{6}$

$$t_2 = \frac{d/3}{3} = \frac{d}{9} \Rightarrow t_3 = \frac{d/3}{6} = \frac{d}{18}$$

$$\text{Average Velocity} = = \frac{d}{\frac{d}{6} + \frac{d}{9} + \frac{d}{18}} = \frac{18}{6} = 3\ m/s$$

Q.2 (B)

$$x = 5\ \sin 10t \Rightarrow v_x = \frac{dx}{dt} = 50\ \cos 10t$$

$$y = 5\cos 10t \Rightarrow v_y = \frac{dy}{dt} = -50\sin 10t$$

$$V^2_{net} = V_x^{\ 2} + v_y^{\ 2}$$

$$v_{net} = \sqrt{(50)^2(\sin^2 10t + \cos^2 10t)} = 50\ m/sec$$

Q.3 (B)

$$v = t^2 - t = t\,(t\text{-}1)$$

$$\therefore\ a = \frac{dv}{dt} = 2t - 1$$

Motion is consider as Retards

when V & a are in opposite Direction

Case - 1

If v > 0 then a < 0

But $t^2 - t > 0$, t > 1

and a > 0 for t > 1

so not Possible

Case - 2

v < 0, a > 0

$t^2 - t < 0,\ 2t - 1 > 0$

$$t \in (0,1),\ t > \frac{1}{2}$$

$$\frac{1}{2} < t < 1$$

Q.4 (B)

Stone is dropped

so time taken by stone to reach the bottom of the wall t_1

$$\therefore\ h = \frac{1}{2}gt_1^{\ 2}$$

$$= t_1 = \sqrt{\frac{2h}{g}}\ -(i)$$

time taken by sound to comes from bottom to upper

end $t_2 = \dfrac{h}{v}$ ...(ii)

$\therefore$ Total time $= t_1 + t_2 = \sqrt{\frac{2h}{g}} + \frac{h}{v}$

Q.5 (A)

distance Travelled by (first ball)

$S = ut + \frac{1}{2}at^2$

$= 5\times2 + \frac{1}{2}\times10\times2^2 = 30\,m$

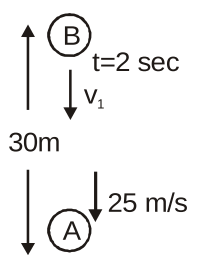
Relative Method

Velocity of first ball after 2 sec.

V = u + at

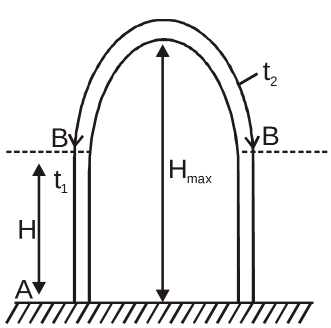
V = 5 + 10 × 2 = 25

$t = \frac{30}{v_1 - 25} = 2\,sec$



$30 = 2v_1 - 50 \Rightarrow v_1 = 40\,m/s$

Q.6 (D)



$H = ut_1 - \frac{1}{2}gt_1^2$ ....(i)

v = u + at

$u = g\left(\frac{t_1 + t_2}{2}\right)$ ....(ii)

From (i) and (ii)

$H = \frac{g}{2}(t_1^2 + t_1t_2) - \frac{1}{2}gt_1^2$

$H = \frac{1}{2}gt_1t_2$

Q.7 (C)

$\therefore H_{max} = \frac{u^2}{2g} \Rightarrow u = \sqrt{2hg}$

Given = 5 m $\Rightarrow$

$H_{max} = 5\,m$

$t = \frac{u}{g} = \frac{\sqrt{2gh}}{g} = \sqrt{\frac{2H}{g}}$

$= \sqrt{\frac{2\times5}{10}} = 1\,sec$

in 1 min = 60 Balls.

Q.8 (D)

v = lnx m/s (Given)

$a = \frac{vdv}{dx} = \frac{1}{x}\ln x$

$\Rightarrow F_{net} = 0$

$\Rightarrow a = 0 \Rightarrow$

x = 1 m

Q.9 (C)

$\vec{F} = 2\sin 3\pi t\hat{i} + 3\cos 3\pi t\hat{j}$

$a = \frac{dv}{dt} = 2\sin 3\pi t\hat{i} + 3\cos 3\pi t\hat{j}$

$\int_0^v dv = 2\int_0^t \sin 3\pi t\, dt\,\hat{i} + 3\int_o^t \cos 3\pi t.dt\hat{j}$

$v = -\frac{2}{3\pi}[\cos 3\pi t]_o^t\hat{i} + \frac{3}{3\pi}[\sin 3\pi t]_o^t\hat{j}$

$\int_o^r dx = \int_o^t\left[\frac{-2}{3\pi}[\cos 3\pi t - 1]\hat{i} + \frac{1}{\pi}\sin 3\pi t\hat{j}\right].dt$

$\vec{r} = -\frac{2}{3\pi}\left[\int_0^t \cos 3\pi t - \int_0^t dt\right]\hat{i} + \frac{1}{\pi}\int_0^t \sin 3\pi t\,\hat{j}dtn$

$= -\frac{2}{(3\pi)^2}[\sin 3\pi t]_o^t\,\hat{i} + \frac{2}{3\pi}t\hat{i} - \frac{1}{3\pi^2}[\cos 3\pi t]_o^t\,\hat{j}$

For t = 1 sec

$\vec{r} = \frac{2}{3\pi}\hat{i} + \frac{2}{3\pi^2}\hat{j}$

Q.10 (B)

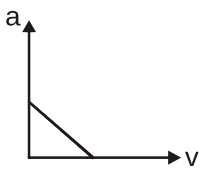
$F = Be^{-ct}$

$$a = \frac{B}{m}e^{-ct} \quad \Rightarrow \int_0^v dv = \int_o^t \frac{B}{m}e^{-ct}dt$$

$$v = -\frac{B}{mc}\left[e^{-ct} - 1\right] \Rightarrow \text{At } t = \infty \quad v = \frac{B}{mc}$$

Q.11 (D)

Q.12 (D)



From graph

$a = -AV + B$

$$\frac{dv}{dt} = -AV + B$$

$$\int \frac{dv}{B - AV} = \int dt$$

$$\Rightarrow \frac{-1}{A}\int \frac{dk}{k} = \int dt \quad (B - AV) = K$$

$$-\ln(B - AV) = At + c$$

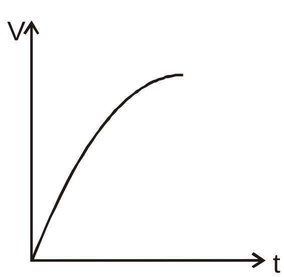
$c = -\ln B$ (When $t = 0, V = 0$)

$$\therefore -\ln(B - AV) = At - \ln B$$

$$t = \frac{1}{A}\ln\left(\frac{B}{B - AV}\right)$$

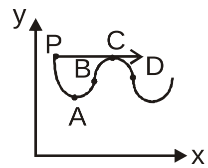
$$e^{At} = \frac{B}{B - AV} \Rightarrow B - AV = Be^{-At}$$

$$\Rightarrow V = \frac{B}{A}\left(1 - e^{-At}\right)$$



Q.13 (C)

Point C



Average Vel. vector is along the x-axis at point 'c' instantaneous vel. vector is along the x-axis.

Q.14 (B)

Area

$$= 0.4 \times 0.2 + 0.4 \times 0.2 + 0.4 \times 0.2 + \frac{1}{2} \times 0.4 \times 0.2 + 0.6 \times 0.2$$

$v_f^2 - v_i^2 = 2ax$

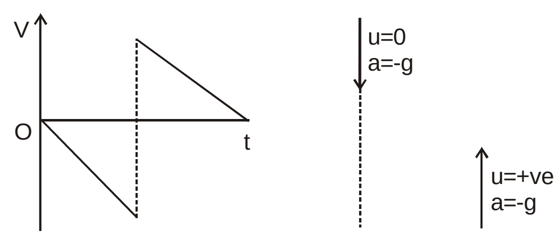
then $v_f^2 - v_i^2 = 2$ Area

$$v_f^2 = 0.8 + (0.8)^2$$

$V_f = 1.2$ m/s

Q.15 (A)

(upward direction is +ve)



Vel. of the particle just before the collision with solid surface euqal to just after polision with solid surface.

Q.16 (D)

Slope of v-t curve gives aceleration

Here slope of P_1 > slope of P_2 $(a_{p_1} > a_{p_2})$

Relative velocity in their motion continousely increases.

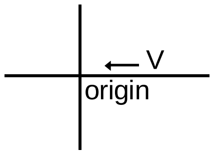
JEE-ADVANCED

MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 (B,C,D)

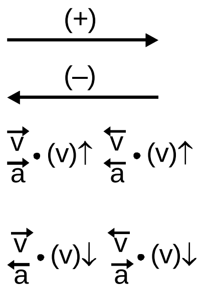
(B) $\Rightarrow a = \dfrac{dv}{dt}$

(C) $\overrightarrow{v}$ $\overleftarrow{a}$ Object is slowing down.

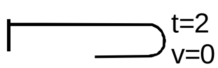
(D) 

the particle is moving towards origin.

Q.2 (C,D)



Q.3 (A,C)
v=10-5t



When v = 0 at t = 2 sec.

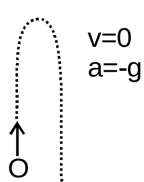
Max displacement $= 10t - \frac{5t^2}{2}$

put t=2 $\Rightarrow$ 20-10=10m
Distance traveled in first 3 seconds

$$= 10 + \left(0 + \frac{1}{2}\times 5\times (1)^2\right) \Rightarrow = 12.5\,\text{m}$$

Q.4 (A,C)
(A) At the top of the motion v = 0 but a = –g.

(A, B) $a = \frac{dv}{dt}$



(C) If particle is moving with costant velocity

(D) No

Q.5 (A,B,C,D)
$X = \alpha T^2 - \beta t^3$

(A) $0 = \alpha t^2 - \beta t^3 \Rightarrow t = \frac{\alpha}{\beta}$

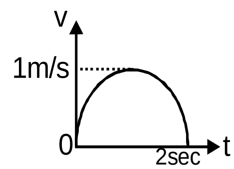
(B) $v = \frac{dx}{dt} = 2\alpha t - 3\beta t^2 \Rightarrow v = 0 \Rightarrow t = \frac{2\alpha}{3\beta}$

(C) $a = \frac{d^2x}{dt^2} = 2\alpha - 6\beta t$
when t=0 $\Rightarrow$ a=2α ; v=0

(D) Acceleration at $t = \frac{\alpha}{3\beta}$; a = 0

$\therefore$ net force = 0

Q.6 (C)

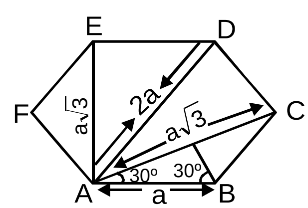


Area $= \frac{\pi(1)^2}{2} = \frac{\pi}{2}\text{m}$

Av velocity $= \frac{\pi}{2\times 2} = \frac{\pi}{4}\text{m/s}$

Q.7 (A,B,C,D)
(a) At T (velocity changes its direction)
(b) slope constant
(c) Upper area = Lower area
(d) Initial speed = final speed.

Q.8 (A,C,D)



(A) A to F

Average velocity $= \frac{\text{Total Desplacement}}{\text{Total time}}$

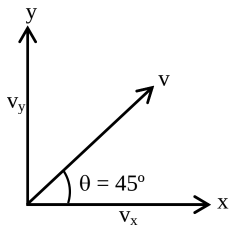
$= \frac{a}{5a/v} = \frac{v}{5}$

(B) A to D $= \frac{2a}{3a/v} = \frac{2}{3}v$

(C) A to C $= \frac{a\sqrt{3}}{2a/v} = \frac{v\sqrt{3}}{2}$

(D) A to B $= \frac{a}{a/v} = v$

Q.9 (A)



Take x-axis along motion of truck and y-axis vertically upward

$\vec{v}_{ball/boy} = (10\,\hat{j})$ m/s

$\vec{v}_{boy/g} = (10\,\hat{i})$ m/s

$\vec{v}_{ball/g} = (10 + 10)$

Distance of ball from pole

$$= \frac{u^2 \sin 2\theta}{g} = \frac{(10\sqrt{2})^2 \times \sin 90}{g}$$

= 20 m

Q.10 (A)

$$\text{Maximum height} = \frac{10^2}{2g} = 5\text{ m}$$

Q.11 (C)

At maximum height, velocity of ball is horizontal. Time taken by ball to reach maximum height

$$t_H = \frac{u_y}{g} = \frac{10}{10} = 1\text{ sec}$$

Velocity of truck is

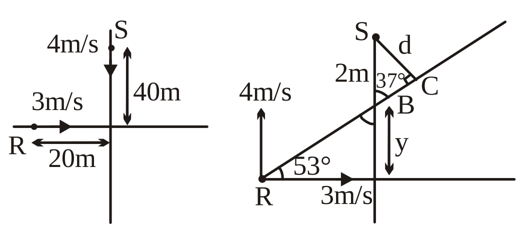
v = u + at

= 10 + 2 × 1 = 12 m/s

Q.12 (C)

$$V_{R/S} = \sqrt{3^2 + 4^2} = 5\text{ m/s}$$

Q.13 (D)



$$\tan 53° = \frac{y}{20}$$

$$y = 20 \times \frac{4}{3} = \frac{80}{3}$$

$$\sin 37° = \frac{d}{40 - \frac{80}{3}} \qquad d = \frac{3}{5}\left[\frac{40}{3}\right] = 8\text{ m}$$

Q.14 (A)

$$\tan 37° = \frac{8}{BC}$$

$$\Rightarrow \quad \frac{3}{4} = \frac{8}{BC}$$

$$\sin 53° = \frac{80}{3AB}$$

$$\Rightarrow \quad t = \frac{\frac{32}{3} + \frac{100}{3}}{5} = \frac{132}{15} = 8.8\text{ sec}$$

Q.15 (B)

$$\text{Distance travelled} = \text{Are} = \frac{1}{2} \times (10 + 5) \times 10 = 75\text{ m}$$

Q.16 (C)

$$ma = |-(10)(1000)\text{kg}|$$

= 10000 N

Q.17 (D)

Q.18 (B)

Particle is at rest when v = 0

⇒ t = 0, and t = 8 sec

Q.19 (C)

$$\text{Rate of change of velocity } \left|\frac{\Delta v}{\Delta t}\right| = \text{is maximum in 4 to 6 s}$$

Q.20 (A)

$$x - (-15) = +\frac{1}{2} \times 2 \times 10$$

x + 15 = 10

x = – 5 m

Q.21 (A)

$$d_{max} = \frac{1}{2}(4.25 + 2) \times 10 = 32.25\text{ m}$$

Q.22 (A)

$$\text{Distance} = 32.25\text{ m} + \frac{1}{2} \times 3.75 \times 10 = 51\text{ m}$$

Q.23 (A)-R ; (B)-P,Q ; (C)-S ; (D)-P,Q,R,S

(A) $v > 0$ and $\frac{dv}{dt} < 0$ or

$v > 0$ and $\frac{dv}{dt} > 0$

(B) $\frac{ds}{dt} > 0$ or

$v > 0$ and $\frac{dv}{dt} > 0$ or

$v < 0$ and $\frac{dv}{dt} < 0$

(C) $\left|\frac{dv}{dt}\right|$ is increasing

(D) $x > 0$, and $v > 0$ or
$x < 0$ and $v < 0$

NUMERICAL VALUE BASED

Q.1 [50]

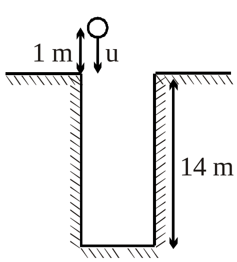
$$a = \frac{v\,dv}{ds}$$

$$\int a\,ds = \int v\,dv$$

$$\frac{v^2}{2} = \frac{1}{2} \times 50 \times 50$$

$\Rightarrow$ $v = 50$ m/s

Q.2 [0030]



For downward motion,
$v^2 = u^2 + 2as = u^2 + 2g \times 15 = u^2 + 300$

$\therefore$ $v = \sqrt{u^2 + 300}$

for upward motion, $u_1 = \frac{v}{2} = \frac{\sqrt{u^2+300}}{2}$

$0^2 = u_1^2 - 2g \times 15$ $\Rightarrow$ $u_1^2 = 300$

$\therefore$ $\frac{u^2 + 300}{4} = 300$

$\therefore$ $u = 30$ ms^{-1}

Q.3 [8 m]
$v^2 = 4 + 4x$
$a = 2, u = 2$

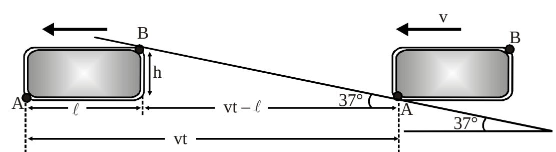
$S = ut + \frac{1}{2}at^2 = 8$ m

Q.4 [30 km]
$4t_1 + 12t_2 = x$ ($t_2 = 2$ hr given)
$4t_1 = x - 24 = 12(t_1 - 1)$

$12 = 8t_1 \Rightarrow t_1 = \frac{3}{2}$ hr

So, $4t_1 = 6$ km and $x = 30$ km

Q.5 [750]



$$\frac{h}{vt - \ell} = \tan 37°$$

$$\Rightarrow \quad v = \frac{\frac{h}{\tan 37°} + \ell}{t} = 7.5 \text{ m/s} = 750 \text{ cm/s}$$

Q.6 [0004]

$$\frac{vdv}{dx} = 4 - 2x$$

$$\int_0^v vdv = \int_0^x (4 - 2x)dx$$

$$\Rightarrow \frac{v^2}{2} = 4x - x^2$$

when $v = 0$, $4x - x^2 = 0$
$x = 0, 4$
$\therefore$ At $x = 4$, the particle will again come to rest.

Q.7 [10]

$s = vt = \frac{1}{2} at^2$ $\quad \therefore\ t = \frac{2v}{a}$

$v_f = at = 2v = 10$ m/s

Q.8 [25]

Average speed = $\frac{\text{distance travelled}}{\text{time taken}}$

$b = \frac{\text{total area}}{\text{total time}} = \frac{10+20}{6} = \frac{30}{6} = 5$ m/s

Average acceleration = $\frac{\text{change in velocity}}{\text{time taken}}$

$C = \frac{10-(-10)}{4} = \frac{20}{4} = 5\ m/s^2$

$bc = (5)(5) = 25\ m^2/s^3$

Q.9 [0075]

$0 = u - 10 \times 1.75$

$u = 17.5\ m/s = 17.5 \times 6 - \frac{1}{2} \times 10 \times 6^2 = 105 - 180 = -75m$

Q.10 [0085]

For max ht.

$0 = u - 10t$

t = 1 sec.

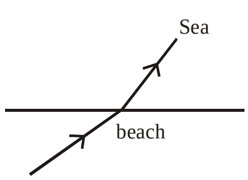
$h_{max} = 10 \times t - \frac{1}{2} \times 10 \times t^2 = 10 - 5 = 5$ m

s_{next} 4 sec $= \frac{-1}{2} \times 10 \times 4^2 = -80$ m

dist. = 80 + 5 = 85 m

KVPY
PREVIOUS YEAR'S

Q.1 (C)

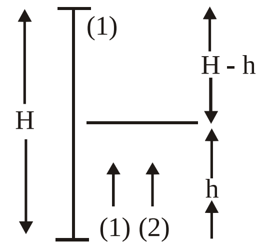


In beach velocity is higher

$\therefore$ beach is rarer and sea is denser medium

So when it go from rarer to denser medium it bend toward normal to reach in minimum time.

Q.2 (C)



$H - h = \frac{1}{2} gt^2$

$h - ut - \frac{1}{2} gt^2$

$H = ut$ ......(1)

$\frac{u^2}{2g} = H$ .....(2)

$t = \frac{u}{2g}$ (from (1) and (2))

$\therefore h = u \times \frac{u}{2g} - \frac{1}{2} g \times \frac{u^2}{4g^2}$

$= \frac{3u^2}{8g} = \frac{3H}{4}$

Q.3 (A)

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$u_x = u$ at (0, b)

$u_y = 0$

$\frac{2x}{a^2}\frac{d}{x} + \frac{2y}{b^2}\frac{dy}{dt} = 0$

Again diff. w.r.t. to time

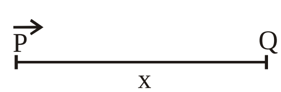
$\frac{2x}{a^2}\frac{d^2x}{xt^2} + \frac{2}{a^2}\left(\frac{dx}{dt}\right)^2 + \frac{2y}{b^2}\frac{d^2y}{dt^2} + \frac{2}{b^2}\left(\frac{dy}{dt}\right) = 0$

acceleration at (0, b) is

$a_y = \frac{-b}{a^2} u^2$

Q.4 (A)

(I) Let initial distance between P and Q is x



at $t_1 = \frac{x}{2} a$ receive the ball.

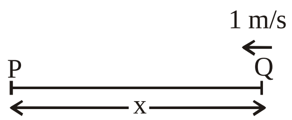
Next ball

$t_2 = \frac{x-5}{2} + 5$

$\Delta t = \frac{5}{2}$

(II) in second case

at t = 0 P throws the ball.



$t_1 = \frac{x}{3}.$

Next ball

$t_2 = \frac{x-5}{3} + 5$

$\Delta t = \frac{10}{3}$

Q.5 (D)

From graph V first increases then decreases Hence a is earliar positive then negative

a = P – qt

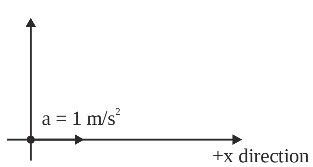
Q.6 (D)

ΔV = const & ΔS increases with time

Q.7 (A)

Magnitude of slope is increasing at point R. Magnitude of slope of displacement-time graph represents speed

Q.8 (2)



$x = \frac{1}{2}at^2$ parabolic for 0 to 4 sec

[at t = 4 sec $x = \frac{1}{2} \times (1)\,(4)^2 = 8m$]

then after (v = 4 m/s)

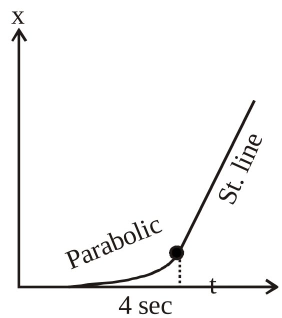
v =4

$\frac{dx}{dt} = 4$

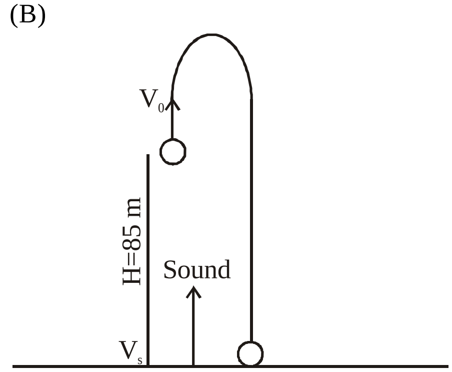
$\int_8^x dx = \int_4^1 4dt$

x – 8 = 4 (t – 4)

x = 4t – 8 (st. line)



Q.9 (B)



Time taken to reach sound after hit $t_s = \frac{H}{V_s}$

$t_s = \left(\frac{85}{340}\right)$ sec; $t_s = 0.25$ sec

For ball time of flight T_f

$T_f + t_s = 5.25$ sec

$T_f = 5.25 - 0.25$

$T_f = 5$ sec

For ball

$V_0 t - \frac{1}{2}gt^2 = -H \Rightarrow V_0(5) - \frac{1}{2}10(5)^2 = -85$

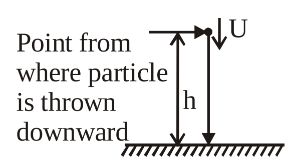
$5V_0 = 40$

$V_0 = 8$ m/s

Q.10 (2)

$-h = -Ut_1 + \frac{-1}{2}\, g\, t_1^2$

$h = Ut_1 + \frac{1}{2}\, g\, t_1^2$ ....(1)



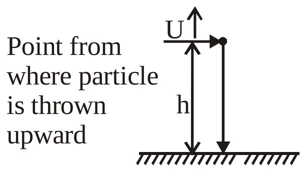
$-h = Ut_2 - \frac{1}{2}\, g\, t_2^2$......(2)

$Ut_1 + Ut_2 = \frac{1}{2} gt_2^2 - \frac{1}{2} g t_1^2$ (from 1 & 2)

$U(t_1 + t_2) = \frac{g}{2} (t_2 - t_1) \times (t_1 + t_2)$

$U = \frac{g}{2} (t_2 - t_1)$

$\therefore$ Max height $= h + \frac{U^2}{2g}$



$= (h) + \frac{U^2}{2g}$

$= Ut_1 + \frac{1}{2} g t_1^{2} + \frac{U^2}{2g}$

$= \frac{g}{2}t_1(t_2 - t_1) + \frac{1}{2}gt_1^2 + \frac{1}{2g}\frac{g^2}{4}(t_2 - t_1)^2$

$= \frac{g}{2}(t_2 - t_1)t_1 + \frac{gt_1^2}{2} + \frac{g}{8}(t_2 - t_1)^2$

$= \left(\frac{g}{2}\right)\{(t_2t_1 - t_1^2) + t_1^2 + \frac{(t_2 - t_1)^2}{4}\}$

$= \left(\frac{g}{2}\right)\frac{\{4t_2t_1 - 4t_1^2 + 4t_1^2 + t_2^2 + t_1^2 - 2t_1t_2\}}{4}$

$= \left(\frac{g}{2}\right)\frac{\{t_1^2 + t_2^2 + 2t_1t_2\}}{4}$

$= \left(\frac{g}{2}\right)\frac{(t_1 + t_2)^2}{4}$

$= \boxed{\frac{g}{8}(t_1 + t_2)^2}$

Q.11 (A)

$v^2 = v^2 - 2gh \rightarrow$ parabola

Q.12 (C)

$f = 8 - 2t^2 = m\frac{dv}{dt}$

$\frac{2}{3}\frac{dv}{dt} = 8 - 2t^2$

$\int dv = \frac{3}{2}\int(8 - 2t^2)$

$\Rightarrow \quad v = \frac{3}{2}\left[8t - \frac{2t^3}{3} + C\right]$ ....(1)

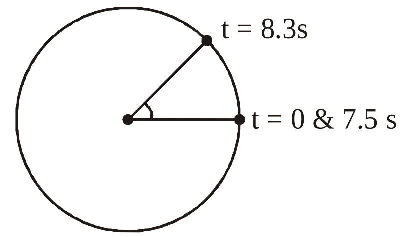
On putting at t = –2 sec v = –15 sec

$\Rightarrow$ C = 2/3

F is zero again at t = 2 sec putting t in (A)

So v' = 17 m/s

Q.13 (D)



$\Delta t = 8.3 - 7.5 = 0.8$ sec

$T = \frac{2\pi}{\omega} = \frac{2 \times 3.14}{\omega} = 1.5$

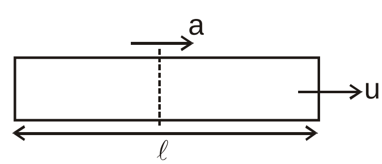
$\omega = \frac{2 \times 3.14}{1.5}$

$\theta = \omega\Delta t = \frac{2 \times 3.14}{1.5} \times 0.8 \approx 3.14 \approx \pi$ rad

So, displacement = 2R = 2m

JEE MAIN
PREVIOUS YEAR'S

Q.1 (2)



$\therefore v^2 = u^2 + 2\ell$

& $v^2_{middlev} = u^2 + 2a\frac{\ell}{2}$

$\therefore v^2_{middle} = u^2 + al$

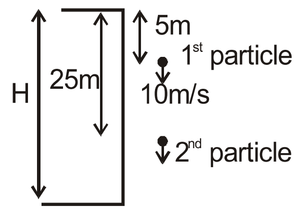
$= u^2 + \left(\frac{v^2 - u^2}{2}\right)$

$= \frac{v^2 - u^2}{2}$

$\therefore v_{middle} = \sqrt{\frac{v^2 + u^2}{2}}$

Q.2 [45 m]

At the instant 2^{nd} particle is dropped 1^{st} particle is moving at 10 m/s & has moved for time 1s.



Time for particles to meet, $\Delta t = \frac{S_{rel}}{V_{rel}} = \frac{20}{10} = 2s$

$\therefore$ Time taken by first particle to reach ground = 3s H

$= \frac{1}{2}g(3)^2 = 45m$

Q.3 (1)

$F = -\alpha x^2 = ma$

$a = \frac{-\alpha x^2}{m} = v\frac{dv}{dx}$

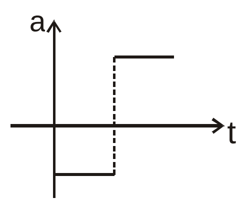
$\int_{v_0}^{0} v\,dv = \int_0^x \frac{\alpha x^2 dx}{m}$

$-\frac{v_0^2}{2} = \frac{\alpha x^3}{3m}$

$x = \left(\frac{3mv_0^2}{2\alpha}\right)^{\frac{1}{3}}$

Q.4 (3)

$v = mt + c$	$v = mt - c$
$\frac{dv}{dt} = m$	$\frac{dv}{dt} = m$



Q.5 (1 or Bonus)

For $0 \le x \le 200$

$v = mx + C$

$v = \frac{1}{5}x + 10$

$a = \frac{vdv}{dx} = \left(\frac{x}{5} + 10\right)\left(\frac{1}{5}\right)$

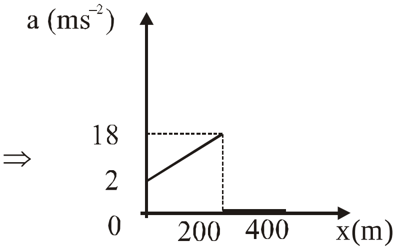
$a = \frac{x}{25} + 2 \Rightarrow$ Straight line till x = 200

for x > 200

v = constant

$\Rightarrow a = 0$

$\Rightarrow$



Hence most approriate option will be (1), otherwise it would be BONUS.

Q.6 (12)

$\vec{a} = \frac{\vec{F}}{m} = \frac{2\hat{i} + 3\hat{j} + 5\hat{k}}{2}$

$= \hat{i} + 1.5\hat{j} + 2.5\hat{k}$

$\vec{\tau} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$= 0 + \frac{1}{2}(\hat{i} + 1.5\hat{j} + 2.5\hat{k})\ (16)$

$= 8\hat{i} + 12\hat{j} + 20\hat{k}$

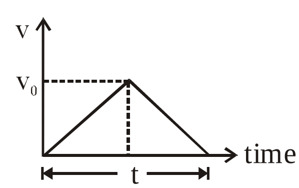
b = 12

Q.7 (3)

$v_0 = \alpha t_1$ and $0 = v_0 - \beta t_2 \Rightarrow v_0 = \beta t_2$

$t_1 + t_2 = t$

$v_0\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = t$



$\Rightarrow v_0 = \frac{\alpha\beta t}{\alpha + \beta}$

Distance = area of v-t graph

$= \frac{1}{2} \times t \times v_0 = \frac{1}{2} \times t \times \frac{\alpha\beta t}{\alpha + \beta} = \frac{\alpha\beta t^2}{2(\alpha + \beta)}$

Q.8 (2)

$v = v_0 + gt + Ft^2$

$\frac{ds}{dt} = v_0 + gt + Ft^2$

$\int ds = \int_0^1 (v_0 + gt + Ft^2)dt$

$s = \left[v_0 t + \frac{gt^2}{2} + \frac{Ft^3}{3}\right]_0^1$

$s = v_0 + \frac{g}{2} + \frac{F}{3}$

Q.9 **(2)**

Option (2) represent correct graph for particle moving with constant acceleration, as for constant acceleration velocity time graph is straight line with positive slope and x-t graph should be an opening upward parabola.

Q.10 [**10**]

$v^2 = u^2 + 2as$

$0 = (10)2 + 2(-a)\left(\frac{1}{2}\right)$

$a = 100\ m/s^2$

$F = ma = (0.1)(100) = 10\ N$

Q.11 **(3)**

$$v = -\left(\frac{v_0}{x_0}\right)x + v_0$$

$$a = \left[-\left(\frac{v_0}{x_0}\right)x + v_0\right]\left[-\frac{v_0}{x_0}\right]$$

$$a = \left(\frac{v_0}{x_0}\right)^2 \qquad \frac{v_0^2}{x_0}$$

Q.12 (4)
Q.13 (1)
Q.14 (3)
Q.15 (3)
Q.16 (2)
Q.17 (3)
Q.18 (3)
Q.19 (1)
Q.20 (3)
Q.21 (4)
Q.22 [30]
Q.23 [1]

$y = mx + C$

$v^2 = \frac{20}{10}x + 20$

$v^2 = 2x + 20$

$2v\frac{dv}{dx} = 2$

$\therefore\ a = v\frac{dv}{dx} = 1$

Q.24 [12]

JEE-ADVANCED PREVIOUS YEAR'S

Q.1 (B)

$$m = \frac{4\pi R^2}{3} \times \rho$$

$$\ell n(m) = \ell n\left(\frac{4\pi}{3}\right) + \ell n(\rho) + 3\ell n(R)$$

$$0 = 0 + \frac{1}{\rho}\frac{d\rho}{dt} + \frac{3}{R}\frac{dR}{dt}$$

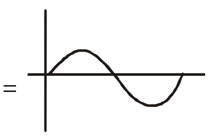
$$\Rightarrow \left(\frac{dR}{dt}\right) = v \propto -R \times \frac{1}{\rho}\left(\frac{d\rho}{dt}\right)$$

$v \propto R$

Q.2 [2.00]

$n = 10^{-3}\ kg \quad q = 1C \quad t = 0$

$E = E_0 \sin \omega t$ =



Force on particle will be

$F = qE = qE_0 \sin \omega t$

at v_{max}, a, F = 0 $\qquad qE_0 \sin \omega t = 0$

$F = qE_0 \sin\omega t$

$$\frac{dv}{dt} = q\frac{E_0}{m}\sin \omega t$$

$$\int_0^v dv = \int_0^{\pi/\omega} \frac{qE_0}{m}\sin \omega t\, dt$$

$$v - 0 = \frac{qE_0}{m\omega}\left[-\cos \omega t\right]_0^{\pi/\omega}$$

$$v - 0 = \frac{qE_0}{m\omega}\left[(-\cos\pi) - (-\cos 0)\right]$$

$$v = \frac{1\times 1}{10^{-3}10^{3}} \times 2 = 2\,m/s$$

EXERCISES

ELEMENTARY

Q.1 (2)

$$\text{Time} = \frac{\text{Total length}}{\text{Relative velocity}} = \frac{50+50}{10+15} = \frac{100}{25} = 4 \text{ sec}$$

Q.2 (1)

Effective speed of the bullet

= speed of bullet + speed of police jeep

= 180 m/s + 45 km/h = (180 + 12.5) m/s = 192.5 m/s

Speed of thief 's jeep = 153 km/h = 42.5 m/s

Velocity of bullet w.r.t thief 's car = 192.5 – 42.5 = 150 m/s

Q.3 (1)

As the trains are moving in the same direction. So the initial relative speed $(v_1 - v_2)$ and by applying retardation final relative speed becomes zero.

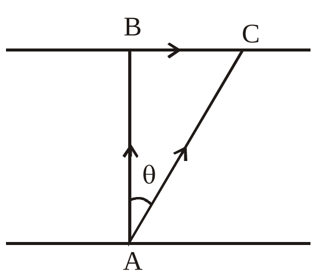
From $v = u - at \Rightarrow 0 = (v_1 - v_2) - at \Rightarrow t = \dfrac{v_1 - v_2}{a}$.

Q.4 (3)

Q.5 (1)

Q.6 (2)

Q.7 (3) Given $\overrightarrow{AB}$ = Velocity of boat = 8 km/hr



$\overrightarrow{AC}$ = Resultant velocity of boat = 10 km/hr

$\overrightarrow{BC}$ = Velocity of river = $\sqrt{AC^2 - AB^2}$

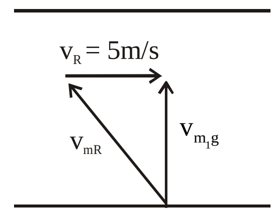
$= \sqrt{(10)^2 - (8)^2} = 6 \text{ km/hr}$

Q.8 (2)

The relative velocity of boat w.r.t. water

$= v_{boat} - v_{water} = (3\hat{i} + 4\hat{j}) - (-3\hat{i} - 4\hat{j}) = (6\hat{i} + 8\hat{j})$

Q.9 (2)



$$\frac{60m}{5} = \sqrt{v_m^2 - v_2^2}$$

$\Rightarrow 144 + 5^2 = v_m^2$

$\Rightarrow v_m = 13 \text{ m/s}$

Q.10 (4)

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 (2)

Total Length of 2 trains = 50 + 50 = 100

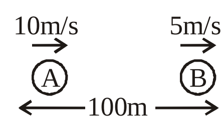
Velocity $V_1 = 10$

$V_2 = 15$

$V_1 + V_2 = 25$

$\text{time} = \dfrac{100}{25} = 4 \text{ sec}$

Q.2 (3)



$V_{AB} = 10 - 5 = 5 \text{ m/s}$

$t = \dfrac{100}{5} = 20 \text{ sec}$.

Q.3 (3)

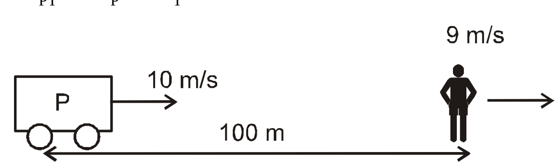
$\upsilon_1 = 50 - gT$

$\upsilon_2 = -50 - gT$

$v_r = \upsilon_1 - \upsilon_2 = 100$ m/sec

Q.4 (4)

$\vec{V}_{PT} = \vec{V}_P - \vec{V}_T = 10 - 9 = 1$ m/s



So time taken $t = \dfrac{100}{\vec{V}_{PT}} = \dfrac{100}{1} = 100$ sec.

Q.5 (4)

Relative velocity between either car (1^{st} or 2^{nd}) and 3^{rd} car = u + 30

where u = velocity of 3^{rd} car

Relative Displacement = 5 km

Time interval = 4 min.

$\therefore u + 30 = \dfrac{5}{4}$ km/min

$= \dfrac{5 \times 60}{4}$ km/h.= 75 $\Rightarrow$ u = 45 km/h.

Q.6 (4)

$V_{rel} = \dfrac{S_{rel}}{t} = \dfrac{1000}{100} = 10$ m/s.

$\therefore V_S - V_B = 10$

$\Rightarrow V_S = 10 + V_B = 10 + 10 = 20$ m/s. **Ans.**

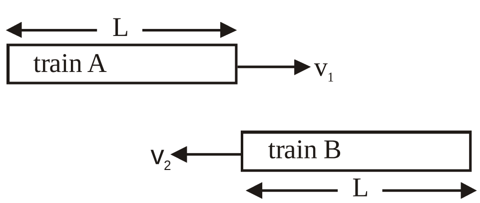
Q.7 (2)

We have, $S_{rel} = u_{rel}t + \dfrac{1}{2}a_{rel}\, t^2$

$\Rightarrow 0 = ut - \dfrac{1}{2}(a + g)\, t^2$

$\Rightarrow a = \dfrac{2u}{t} - g = \dfrac{2u - gt}{t}$

Q.8 (3)



$t_1 = 3 = \dfrac{2L}{v_1 + v_2}$

$\Rightarrow v_1 + v_2 = \dfrac{2L}{3}$ ....(i)

$t_2 = 2.5 = \dfrac{2L}{1.5v_1 + v_2}$ $\quad 1.5\, v_1 + v_2 = \dfrac{4L}{5}$

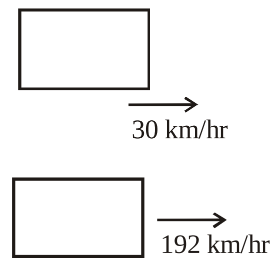
...........(ii)

by (i) and (ii)

$v_1 = \dfrac{4L}{15}$; $v_2 = \dfrac{2L}{5}$

Now, $t_3 = \dfrac{2L}{|v_1 - v_2|} = \dfrac{2L}{2L/15} = 15$ sec.

Q.9 (1)



$\dfrac{25}{3}$ m/s = 30 km/h

Muzzle speed = velocity of bullet w.r.t. revolver

= velocity of bullet w.r.t. van

$150 = V_b - V_v$

$150 = V_b - \dfrac{25}{3} \Rightarrow V_b = \dfrac{475}{3}$ m/s w.r.t. ground

Now speed with which bullet hit thief's car

= velocity of bullet w.r.t car

$= V_{bc} = V_b - V_c$

$= \dfrac{475}{3} - \dfrac{160}{3} = \dfrac{315}{3} = 105$ m/s **Ans.**

Q.10 (4)

Initial relative velocity

$u_r = 50 - (-50) = 100$

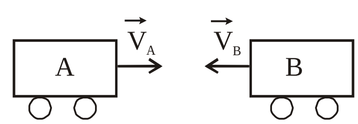
$a_r = 20 - (20) = 0$

$s_r = \upsilon_r t + \dfrac{1}{2}\, a_r t^2$

$100 = 100\,t$ $\quad t = 1$ hr

$s_A = 50\,(1) + \dfrac{1}{2}\,(20)\,(1)^2 = 60$ km.

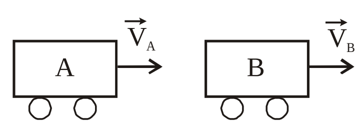
Q.11 (1)



In opposite direction

$\vec{V}_{AB} = \vec{V}_A + \vec{V}_B$

$9 = \vec{V}_A + \vec{V}_B$ .... (1)



In same direction -

$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$

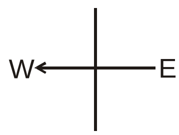
$1 = \vec{V}_A - \vec{V}_B$ .... (2)

From equation (1) & (2)

$\vec{V}_A$ = 5 m/s

$\vec{V}_B$ = 4 m/s

Q.12 (1)



$\vec{V}_A = -500\,\hat{i}$

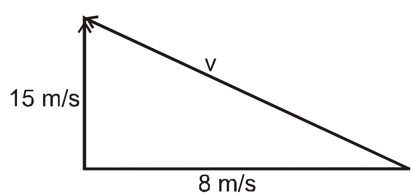
$\Rightarrow \vec{V}_{GA} + \vec{V}_A = V_{GAS}$

$\vec{V}_{GA} = 1500\,\hat{i}$

$\Rightarrow 1500\,\hat{i} - 500\,\hat{i} = 1000\,\hat{i}$

Q.13 (2)

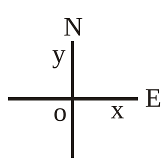
If v = actual velocity



v = $\sqrt{15^2 + 8^2}$

= 17 m/s.

Q.14 (4)



$\vec{V}_r = 50\,(-\hat{j}) - 50\,\hat{i} = 50\,(-\hat{i} - \hat{j})$ i.e., in south west

Q.15 (4)

$\vec{V}_{12} = \vec{V}_1 - \vec{V}_2$

$|\vec{V}_{12}| = \sqrt{V_1^2 + V_2^2 - 2V_1V_2\cos\theta}$

If cos θ = – 1

$|\vec{V}_{12}|_{max} = \sqrt{V_1^2 + V_2^2 + 2V_1V_2}$

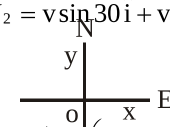
$|\vec{V}_{12}|_{max} = (V_1 + V_2)$

So $|\vec{V}_{12}|$ is maximum when $\cos\theta = -1$ and $\theta = \pi$

Q.16 (3)

$\vec{V}_1 = 10\,\hat{i}$

$\vec{V}_2 = v\sin 30\,\hat{i} + v\cos 30\,\hat{j} = \frac{\upsilon}{2}\hat{i} + \frac{\upsilon\sqrt{3}}{2}\hat{j}$



$\vec{V}_2 - \vec{V}_1 = \left(\frac{\upsilon}{2} - 10\right)\hat{i} + \frac{\upsilon\sqrt{3}}{2}\hat{j} = \frac{\upsilon\sqrt{3}}{2}\hat{j}$

$\therefore \frac{\upsilon}{2} - 10 = 0$ or

$\upsilon = 20$

Q.17 (4)

$\vec{V}_1 = \hat{i} + \sqrt{3}\,\hat{j}$ $\tan\theta_1 = \sqrt{3}$

i.e., $\theta_1 = 60$.

$\vec{V}_2 = 2\hat{i} + 2\,\hat{j}$ $\tan\theta_2 = 1$

i.e., $\theta_2 = 45^o$ $\theta_1 - \theta_2 = 15^o$

Q.18 (2)

Q.19 (1)

$\vec{V}_r = \upsilon_y\, j$

$\vec{\upsilon}_m = 5\hat{i}$

$\vec{V}_r - \vec{V}_m = (-5)\,\hat{i} + \upsilon_y\,\hat{j}$

$\tan\theta = 1 = \frac{\upsilon_y}{5}$

so υ_y = 5 km/hr

Q.20 (2)

$\vec{V}_r = 10\,\hat{j}$

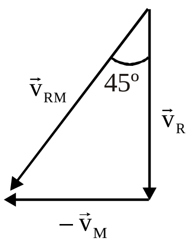
$\vec{V}_c = \upsilon\,\hat{i}$

$\vec{V}_r - \vec{V}_c = 10\,\hat{j} - \upsilon\hat{i}$

$|\vec{V}_r - \vec{V}_c| = \sqrt{10^2 + \upsilon^2} = 20$

$\upsilon = 10\sqrt{3}$

Q.21 (1)

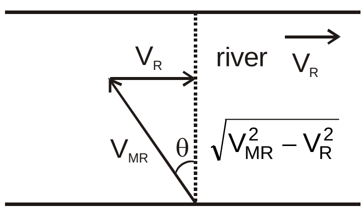


$\vec{v}_{RM} = \vec{v}_R - \vec{v}_m$

so $v_R = v_M$

Q.22 (2)

15 min = 1/4 hr.



Q.23 (2)

$T_0 = \frac{d}{V} + \frac{d}{V} = \frac{2d}{V}$

$T = \frac{d}{V+u} + \frac{d}{V-u} = \frac{2Vd}{V^2 - u^2}$

$T = \frac{V^2 T_0}{V^2 - u^2} = \frac{T_0}{1 - \frac{u^2}{V^2}}$

$\frac{T}{T_0} = \frac{1}{1 - \frac{u^2}{V^2}}$

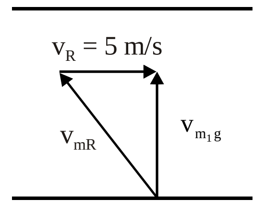
Q.24 (3)

viewing the motion from river frame ; both boats will reach the ball simultaneously

Q.25 (1)

Due north will take him cross in shortest time.

Q.26 (2)



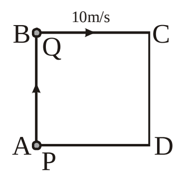
$V_{MR} = \frac{60}{5} = 12$

$\vec{V}_{MR} = \vec{V}_M - \vec{V}_R$

$\vec{V}_M = \vec{V}_{MR} + \vec{V}_R$

$= \sqrt{12^2 + 5^2} = 13 ms^{-1}$

Q.27 (2)



a = 8 m

They meet when Q displace 8 × 3 m

more than p ⇒ relative displacement = relative velocity × time.

8 × 3 = (10 – 2) t

t = 3 sec

JEE-ADVANCED
OBJECTIVE QUESTIONS

Q.1 (D)

Relative velocity between either car (1st or 2nd) and 3rd car = u + 30

where u = velocity of 3rd car

Relative Displacement = 5 km

Time interval= 4 min.

$\therefore u + 30 = \frac{5}{4}$ km/min

$= \frac{5 \times 60}{4}$ km/h.= 75 ⇒ u = 45 km/h.

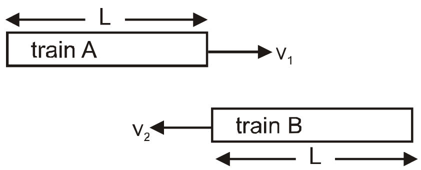
Q.2 (B)

We have, $S_{rel} = u_{rel}t + \frac{1}{2} a_{rel} t^2$

$\Rightarrow 0 = ut - \frac{1}{2}(a + g) t^2$

$\Rightarrow a = \frac{2u}{t} - g = \frac{2u - gt}{t}$

Q.3 (C)



$t_1 = 3 = \dfrac{2L}{v_1 + v_2} \Rightarrow v_1 + v_2 = \dfrac{2L}{3}$ ....(i)

$t_2 = 2.5 = \dfrac{2L}{1.5v_1 + v_2}$

$1.5\,v_1 + v_2 = \dfrac{4L}{5}$ ....(ii)

by (i) and (ii)

$v_1 = \dfrac{4L}{15}\ ;\ v_2 = \dfrac{2L}{5}$

Now, $t_3 = \dfrac{2L}{|v_1 - v_2|} = \dfrac{2L}{2L/15} = 15$ sec.

Q.4 (C)

No. of taxi $= \dfrac{240}{10} = 24$ but when 24th start motion it reach the destination so it will meet 23 only.

Q.5 (A)

Let t be time when car catches the motor cycle

So $vt = 100 + \dfrac{1}{2}at^2 \Rightarrow at^2 - 2vt + 200 = 0$

t should be real $D \geq 0$

$4v^2 - 800a \geq 0$

$\dfrac{4 \times 20 \times 20}{800} \geq a \qquad \Rightarrow \quad a \leq 2$ m/s

Q.6 (C)

	A	B	BA
	$u_x = 0$	$u_x = u$	$u_x = u$
	$\therefore$ horizontal		
↓ +y	$u_y = 0$	$u_y = 0$	0
	$a_x = 0$	$a_x = 0$	0
	$a_y = g$	$a_y = g$	0

Q.7 (D)

At t = 3 sec, 1st stone will have speed of 30 m/s

$h_1 = \dfrac{1}{2} \times 10 \times 9 = 45$ m; $h_2 = \dfrac{1}{2} \times 10 \times 1^2 = 5$ m

$h_1 - h_2 = 40$ m

Q.8 (B)

With respect to lift initial speed $= v_0$

acceleration $= -2g$

displacement $= 0 \qquad \therefore S = ut + \dfrac{1}{2}at^2$

$0 = v_0T' + \dfrac{1}{2} \times 2g \times T'^2$

$\therefore T' = \dfrac{v_0}{g} = \dfrac{1}{2} \times \dfrac{2v_0}{g} = \dfrac{1}{2}T$

Q.9 (C)

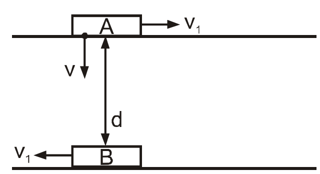
Relative to the person in the train, acceleration of the stone is 'g' downward, a (acceleration of train) backwards.

According to him :

$x = \dfrac{1}{2}at^2$, $\qquad Y = \dfrac{1}{2}gt^2$

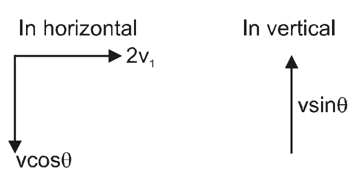
$\Rightarrow \dfrac{X}{Y} = \dfrac{a}{g} \Rightarrow Y = \dfrac{g}{a}x \Rightarrow$ straight line.

Q.10 (D)



velocity of particle w.r.t. B

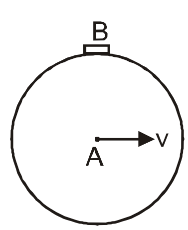
Let particle is projected at angle θ with horizontal



So, path observed by B is parabolic (projectile motion).

Q.11 (C)

Relative to B, particle A is moving around it with constant speed v.



So, w.r.t. B, A appears to be move in circle motion.

Q.12 (C)

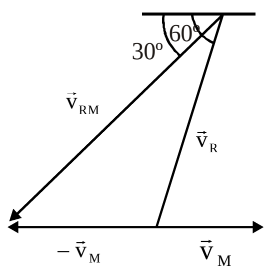
The trajectory can be straight line.

Q.13 (A)

$$v_{bt} = v_b - v_t$$

(A) If train is accelerating then the ball will cover less distance with respect to train in later part of motion.

Q.14 (A)



$$\vec{v}_{RM} = \vec{v}_R - \vec{v}_m$$

$$v_R \cos 60^o + v_M = v_{RM} \cos 30^o$$

(1) $v_R \sin 60^o = v_{RM} \sin 30^o$

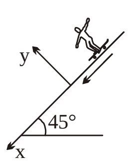
(2) Putting this in equation (1)

$$v_R \frac{1}{2} + 25 = v_R \sqrt{3} \cdot \frac{\sqrt{3}}{2}$$

$$\Rightarrow v_R\left(\frac{3}{2} - \frac{1}{2}\right) = 25$$

$$v_R = 25 \text{ m/s}$$

Q.15 (A)

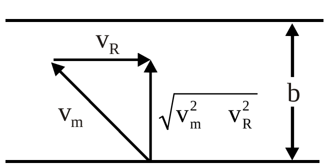


$$V_{ball,\, bay} = \hat{j}$$

Q.16 (B)

$$T_1 = \frac{2b}{\sqrt{v_m^2 - v_R^2}}$$

(1)



$$T_2 = \frac{b}{v_m + v_R} + \frac{b}{v_m - v_R}$$

$$\Rightarrow T_2 = \frac{2bv_m}{v_m^2 - v_R^2}$$

$$\Rightarrow \frac{2b}{v_m^2 - v_R^2} = \frac{T_2}{v_m} \qquad (2)$$

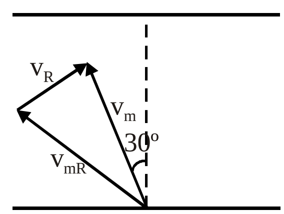
$$T_3 = \frac{2b}{v_m} \qquad (3)$$

squaring (1) $T_1^2 = \frac{2b}{v_m^2 - v_R^2} \times 2b$

$$\Rightarrow \boxed{T_1^2 = T_2 \cdot T_3}$$

Q.17 (C)

$$\vec{v}_{mR} = \vec{v}_m - \vec{v}_R$$



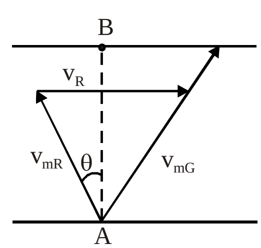
$$v_m = 5\hat{i} + 5\sqrt{3}\left((-\sin(\theta+30)\hat{i} + \cos(\theta+30)\hat{j}\right)$$

$$v_m = \left(5 - 5\sqrt{3}\sin(\theta+30)\right)\hat{i} + 5\sqrt{3}\cos(\theta+20)\hat{j}$$

$$\tan 30^o = \frac{5 - 5\sqrt{3}\sin(\theta+30)}{5\sqrt{3}\cos(\theta+30)} \Rightarrow \theta = -30^o$$

Q.18 (B)

For least drift $\sin\theta = \frac{v_{mR}}{v_R} = \frac{5}{6}$



$$x = \frac{d}{v_{mR}\cos\theta}\left(v_R - v_{mR}\sin\theta\right)$$

Q.19 (B)

$$\vec{v}_{AB} = v(\hat{j}+\hat{i}) = \vec{v}_A - \vec{v}_B \quad (1)$$

$$\vec{v}_{BC} = v(-\hat{i}+\hat{j}) = \vec{v}_B - \vec{v}_C \quad (2)$$

adding (1) & (2)

$$\vec{v}_A - \vec{v}_C = 2v\,\hat{j}$$

Hence towards North

Q.20 (D)

Let h = height of escalator

v = speed of escalator

u = speed of walking of man

t_3 = time taken to reach tower if man walks up on a moving escalator.

$h = vt_1 = ut_2 = (v+u)\,t_3$

$$\Rightarrow t_3 = \frac{h}{v+u} \;;\; v = \frac{h}{t_1} \;;\; u = \frac{h}{t_2}$$

$$\therefore t_3 = \frac{1}{\frac{1}{t_1}+\frac{1}{t_2}} = \frac{t_1 t_2}{t_1+t_2} \textbf{ Ans.}$$

Q.21 (B)

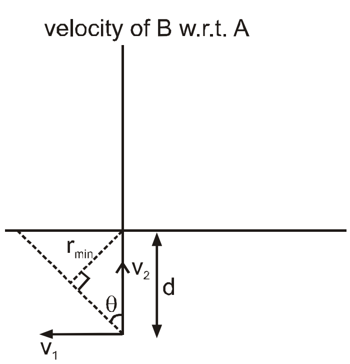
A and B collide if $\dfrac{\vec{r}_B - \vec{r}_A}{|\vec{r}_B - \vec{r}_A|} = \pm\dfrac{\vec{V}_B - \vec{V}_A}{|\vec{V}_B - \vec{V}_A|}$

$$= \frac{4\hat{i}+4\hat{j}}{4\sqrt{2}} = \pm\frac{(x-3)\,\hat{i}\text{-}4\hat{j}}{|(x-3)^2+4^2|}$$

By compare

$x - 3 = -4 \;\Rightarrow\; x = -1.$

Q.22 (C)



$\tan\theta = \dfrac{v_1}{v_2}$

$r_{min} = d\sin\theta$

$= d\,.\dfrac{v_1}{\sqrt{v_1^2+v_2^2}}\,.$

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MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 (B,C,D)
Q.2 (B,C)
Q.3 (A,B)
Q.4 (A,B,D)
Q.5 (A,B,C)
Q.6 (C,D)
Q.7 (A,D)
Q.8 (A,B,D)
Q.9 (A, C)
Q.10 (B, D)
Q.11 (B)

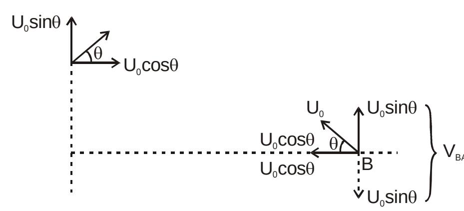
Q.12 (B)

The path of a projectile as observed by other projectile is a straight line.

$V_A = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\,\hat{j}\,.\; V_{AB} = (2u\cos\theta)\,\hat{i}$

$V_B = -u\cos\theta\,\hat{i} + (u\sin\theta - gt)\,\hat{j}\,.$

$a_{BA} = g - g = 0$



The vertical component $u_0\sin\theta$ will get cancelled. The relative velocity will only be horizontal which is equal to $2u_0\cos\theta$.
Hence B will travel horizontally towards left w.r.t. A with constant speed $2u_0\cos\theta$ and minimum distance will be h.

Q.13 (C)

Time to attain this separation will obviously be $\dfrac{S_{rel}}{V_{rel}} = \dfrac{\ell}{2u_0\cos\theta}.$

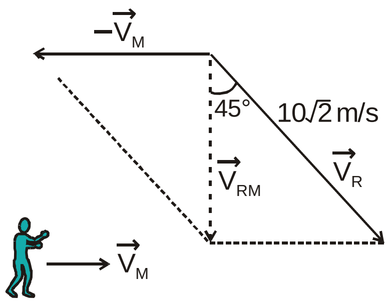
Q.14 (D)

Q.15 (A)

Q.16 (B)

In the first case :

From the figure it is clear that



$\vec{V}_{RM}$ is 10 m/s downwards and

$\vec{V}_M$ is 10 m/s towards right.

In the second case :

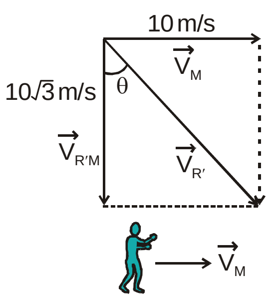
Velocity of rain as observed by man becomes $\sqrt{3}$ times in magnitude.

$\therefore$ New velocity of rain

$\vec{V}_{R'} = \vec{V}_{R'M} + \vec{V}_M$

$\therefore$ The angle rain makes with vertical is

$\tan\theta = \dfrac{10}{10\sqrt{3}}$ or $\theta = 30°$



$\therefore$ Change in angle of rain = 45 – 30 = 15°.

Q.17 (D)

Q.18 (C)

Q.19 (A)

Q.20 (A)

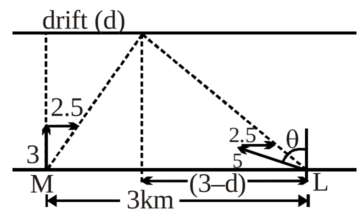
3km = (5 + 3)t + (5 – 3)t

So, $t = \dfrac{3\text{km}}{10 \text{ km/hr}} = \dfrac{3}{10}$ hr. = 18 min.

Q.21 (C)

t_{cross} for Mahiwal : $\dfrac{3\text{km}}{5\text{km/hr}}$ = 36 min.

$$\text{drift}(d) = (2.5 \text{ km/hr})\left(\frac{3\text{km}}{5\text{km/hr}}\right) = \frac{3}{2} \text{ km}$$



t_{cross} for Soni = $\dfrac{3\text{km}}{5\cos\theta}$

$$\text{drift}' = (3-d) = (5\sin\theta - 2.5)\frac{3}{5\cos\theta}$$

$$3 - \frac{3}{2} = \left(5\sin\theta - \frac{5}{2}\right)\frac{3}{5\cos\theta}$$

$$\frac{3}{2} = \left(\frac{2\sin\theta - 1}{2}\right)\frac{3}{\cos\theta}$$

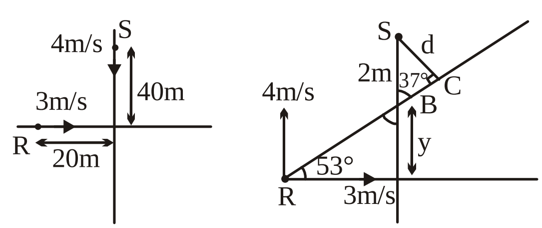
$\cos\theta = 2\sin\theta - 1$ $\Rightarrow \theta = 53°$

So angle with river flow = 90° + 53° = 143°

Q.22 (C)

$V_{R/S} = \sqrt{3^2 + 4^2}$ = 5 m/s

Q.23 (D)



$\tan 53° = \dfrac{y}{20}$

$y = 20 \times \dfrac{4}{3} = \dfrac{80}{3}$

$\sin 37° = \dfrac{d}{40 - \dfrac{80}{3}}$ $\qquad d = \dfrac{3}{5}\left[\dfrac{40}{3}\right]$ = 8 m

Q.24 (A)

$$\tan 37° = \frac{8}{BC}$$

$$\Rightarrow \frac{3}{4} = \frac{8}{BC}$$

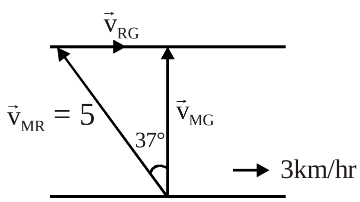
$$\sin 53° = \frac{80}{3AB}$$

$$\Rightarrow \quad t = \frac{\frac{32}{3}+\frac{100}{3}}{5} = \frac{132}{15} = 8.8 \text{ sec}$$

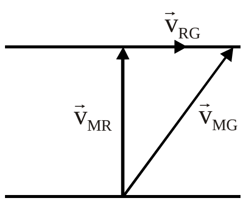
Q.25 (A) Q (B) R (C) S (D) T

Q.26 (A) P (B) R (C) Q

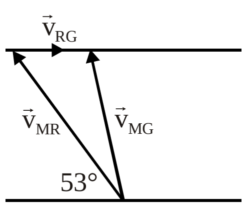
(A)



(B)

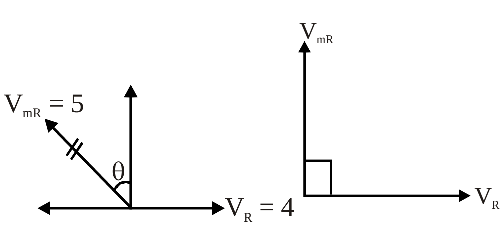


(C)



Q.27 (A)-R ; (B)-S ; (C)-P ; (D)-Q

(A) $V_{mR} \sin\theta = V_R$



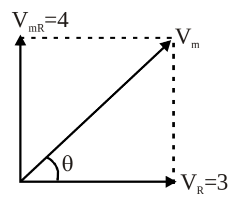
$$\Rightarrow \sin\theta = \frac{4}{5}$$

$\Rightarrow$ θ = 53°
$\Rightarrow$ θ + 90 = 143°

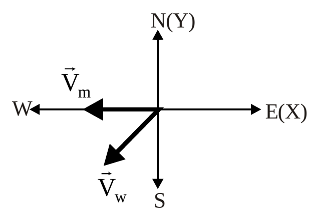
(B) Direction of velocity with the direction of flow = 90°

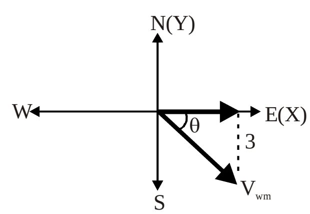
(C) $\tan\theta = \frac{4}{3} \Rightarrow \quad \theta = 53°$

(D) $\vec{V}_w = -3\hat{i} - 3\hat{j}$



$$\Rightarrow \quad \vec{V}_m = -7\hat{i}$$





$$\vec{V}_{wm} = \vec{V}_w - \vec{V}_m$$

$$= -3\hat{i} - 3\hat{j} + 7\hat{i} = 4\hat{i} - 3\hat{j}$$

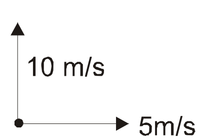
$\tan\theta = \frac{3}{4} \quad \Rightarrow \ \theta = 37° \quad \Rightarrow \ \theta + 90 = 127°$

Q.28 (A) P,Q,R (B) Q, R, S (C) Q,R,S

Q.29 (A) q, (B) q , (C) q, (D) q
In all cases, angle between velocity and net force (in the frame of observer) is in between 0° and 180° (excluding both values, in that path is straight line).

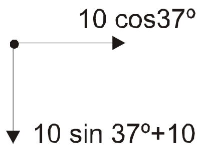
Q.30 (A) q, (B) r (C) p (D) q
(A) V_{BA} = 10 + 10 = 20
so distance b/w B and A in 2sec. = 2 × 20 = 40 m

(B)



$\vec{V}_{BA} = 5\hat{i} - 10\hat{j}$

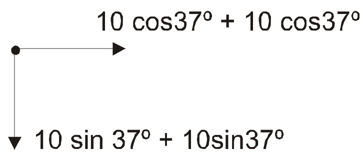
$\Rightarrow \ |V_{BA}| = \sqrt{25+100} = 5\sqrt{5}$

(C) $\vec{V}_{BA}$:- 

so $|\vec{V}_{BA}| = 8\sqrt{5}$

so distance between A and B in 2 sec. = 2 × $8\sqrt{5}$

$= 16\sqrt{5}$

(D) $\vec{V}_{BA}$:- 

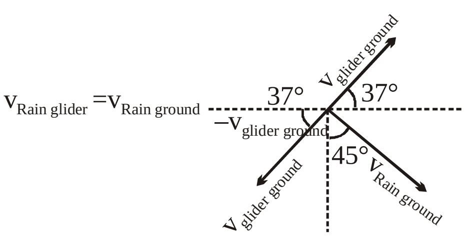
so

$|\vec{V}_{BA}| = 20$

so distance between A and B in 2 sec. = 2 × 20 = 40 m.

NUMERICAL VALUE BASED

Q.1 [0015]

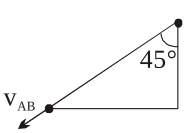


$(v_{\text{glider ground}}) = u$

$u\cos 37° = 12\sqrt{2}\ \sin 45° \quad u = 15$ m/s

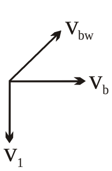
Q.2 [0000]

$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = -20\hat{i} - 20\hat{j}$



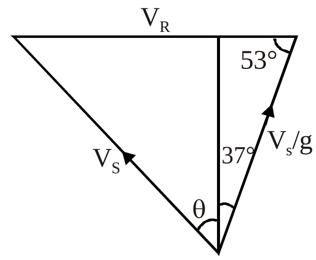
Q.3 [12]

$V_{b\omega}^2 - V_1^2 = V_b^2$



$\Rightarrow \ V_b = 12$ m/s

Q.4 [0016]



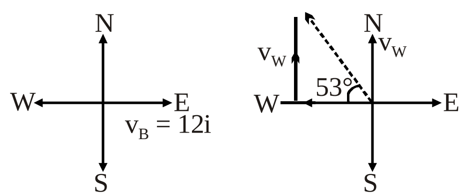
$V_S = V_R$
$53° = \theta + 37°$
$\theta = 16°$]

Q.5 [0001]

$2.5 = \frac{1}{2} \times 10 \times \frac{t^2}{2}$

t = 1 sec.

Q.6 [16]



$v_{WB} = \vec{v}_B - \vec{v}_B$

$\tan 53° = \frac{v_w}{12} = \frac{4}{3}$

$v_w = 16$ ms^{-1}

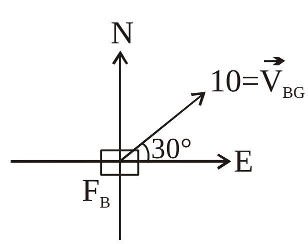
Q.7 [0010]

$3 = \frac{d}{7.5 + 2.5} + \frac{d}{7.5 - 2.5}$

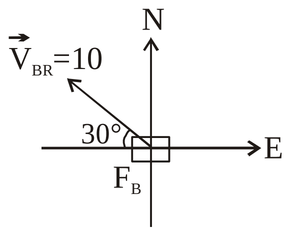
$3 = \frac{d}{10} + \frac{d}{5} \Rightarrow 3 = \frac{d+2d}{10}$

$d = 10$ km

Q.8 [0017]
Ground frame :



River frame :



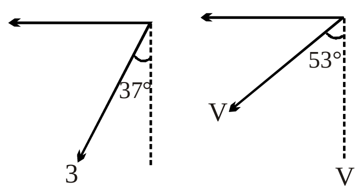
$$\vec{V}_{BG} = \vec{V}_{BR} + \vec{V}_{RG}$$

$$\Rightarrow 5\sqrt{3}\,\hat{i} + 5\hat{j} = -5\sqrt{3}\,\hat{i} + 5\hat{j} + \vec{V}_{RG}$$

$$\therefore\ \vec{V}_{RG} = 10\sqrt{3}\,\hat{i}$$

$\therefore\ V_{RG} = 17.3$ m/s

Q.9 [0007]



$$\frac{3V}{5} = \frac{12}{5}$$

$V = 4$

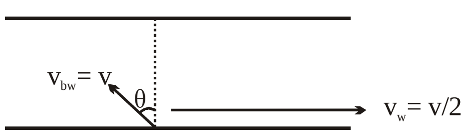
$$\frac{4V}{5} = \frac{9}{5} + V_C = \frac{7}{5}$$

$V_C = 0 + a_C t$

$a_C = 7$

KVPY
PREVIOUS YEAR'S

Q.1 (D)



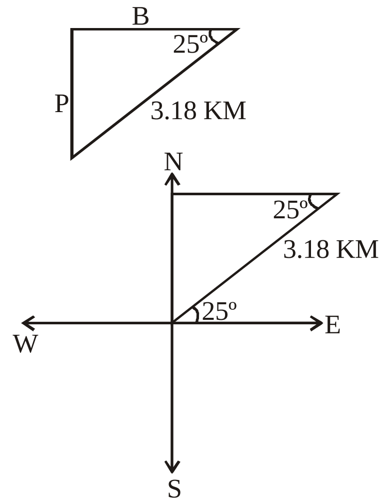
To minimize the drifting

$$\sin\theta = \frac{v_w}{v_{bw}} = \frac{1}{2}$$

$\theta = 30^o$

$90^o + \theta = 120^o$

Q.2 (D)



$$\therefore\ \frac{P}{3.18} = \sin 25^o$$

$P = 3.18 \sin 25^o$

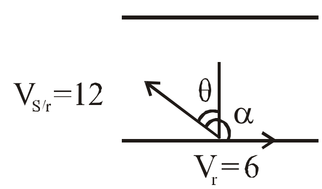
$= \boxed{1.34KM}$ along north

$B = 3.18 \cos 25^o$

$= \boxed{2.88KM} \rightarrow$ along east

JEE-MAIN
PREVIOUS YEAR'S

Q.1 [120]

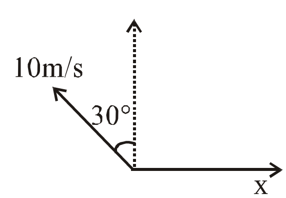


$12\sin\theta = vr$

$\theta = 30°$

$\therefore\ \alpha = 120°$

Q.2 [5]



$10 \sin 30° = x$

$x = 5$ m/s

Q.3 [5]

8β d= 2.4 cm

$$\frac{8\lambda\Delta}{d} = 2.4\text{cm}$$

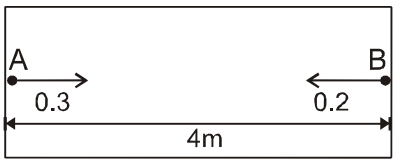
$$\frac{8\times1.5\times c}{0.3\times10^{-3}\times f} = 2.4\times10^{-2}$$

$f = 5\times 10^{14}$ Hz

Q.4 [30]

JEE-ADVANCED
PREVIOUS YEAR'S

Q.1 (2)



consider motion of two balls with respect to rocket
Maximum distance of ball A from left wall =

$$\frac{u^2}{2a} = \frac{0.3\times0.3}{2\times2} = \frac{0.09}{4} \approx 0.02\text{ m}$$

so collision of two balls will take place very near to left wall

For B

$$S = ut + \frac{1}{2}at^2$$

$$-4 = -0.2t - \left(\frac{1}{2}\right)2t^2$$

$\Rightarrow$ $t^2 + 0.2t - 4 = 0$

$\Rightarrow$ $t = \dfrac{-0.2 \pm \sqrt{0.04+16}}{2} = 1.9$

nearest integer = 2s

EXERCISES

ELEMENTARY

Q.1 (3)

$$v_x = \frac{dx}{dt} = 6 \text{ and } v_y = \frac{dy}{dt} = 8 - 10t$$

$$= 8 - 10 \times 0 = 8$$

$$\therefore\ v = \sqrt{v_x^2 + v_y^2} = \sqrt{6^2 + 8^2} = 10\ ms^{-1}$$

Q.2 (4)

$R = \dfrac{u^2 \sin 20}{g}$ $\therefore$ $R \propto u^2$. If initial velocity be doubled then range will become four times.

Q.3 (1)

Range $= \dfrac{u^2 \sin 2\theta}{g}$; when $\theta = 90^o$, R = 0 i.e the body will fall at the point of projection after completing one dimensional motion under gravity.

Q.4 (1)
Direction of velocity is always tangent to the path so at the top of trajectory, it is in horizontal direction and acceleration due to gravity is always in vertically downward direction. It means angle between $\vec{v}$ and $\vec{g}$ are perpendicular to each other.

Q.5 (1)

For vertical upward motion $h = ut - \frac{1}{2}gt^2$

$$5 = (25 \sin\theta) \times 2 - \frac{1}{2} \times 10 \times (2)^2$$

$$\Rightarrow 25 = 50 \sin\theta \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30^o.$$

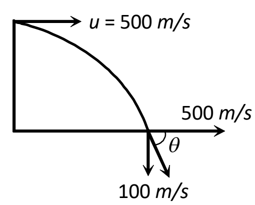
Q.6 (2)

Range is given by $R = \dfrac{u^2 \sin 2\theta}{g}$

On moon $g_m = \dfrac{g}{6}$. Hence $R_m = 6R$.

Q.7 (3)
Due to constant velocity along horizontal and vertical downward force of gravity stone will hit the ground following parabolic path.

Q.8 (1)
Horizontal component of velocity v_x = 500 m/s
and vertical components of velocity while striking the ground.
$v_y = 0 + 10 \times 10 = 100$ m/s



$\therefore$ Angle with which it strikes the ground.

$$\theta = \tan^{-1}\left(\frac{\nu_y}{\nu_x}\right) = \tan^{-1}\left(\frac{100}{500}\right) = \tan^{-1}\left(\frac{1}{5}\right).$$

Q.9 (2)

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 (2)

$V_x = 2at$

$V_y = 2bt$

$V = 2t\sqrt{a^2 + b^2}$

Q.2 (4)
Horizontal Component of velocity
Because there is no acceleration in horizontal Direction

Q.3 (3)
In projectile motion Horizontal accelereation a_x = o &
Vertical acceleration a_y = g =10m/s^2 a_x = o
$a_x = 0$
$a_y = 10$ (down)
$\Rightarrow$ only "C" is correct **Ans**

Q.4 (4)

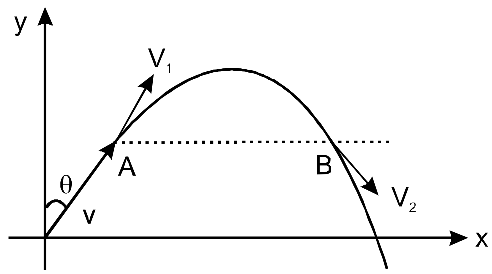
$\theta_1 = \dfrac{5\pi}{36}$

Another angle for same range is the complementary angle θ_2

Then $\theta_2 = \dfrac{\pi'}{2} - \left(\dfrac{5\pi}{36^o}\right)^c = \dfrac{18-5}{36}\pi \Rightarrow \theta_2 = \dfrac{13}{36}\pi$

"D"

Q.5 (1)



Avg. vel. b/w A & B $= \dfrac{\vec{V}_1 + \vec{V}_2}{2}$

($\because$ Acceleration is constant = g)

Now, if $\vec{V}_1 = V_{1x}\,\hat{i} + V_{1y}\,\hat{j}$

Thañ $\vec{V}_2 = V_{1x}\,\hat{i} - V_{1y}\,\hat{j}$ ($\because$ both A & B are at same lavel)

$\therefore \dfrac{\vec{V}_1 + \vec{V}_2}{2} = V_{1x}\,\hat{i} = V\sin\theta\,\hat{i}$ ($\because$ θ is from vertical)

" B "

Q.6 (1)

Gravitational acceleration is constant near the surface of the earth.

Q.7 (2)

$\vec{u}_x = 6\,\hat{i} + 8\,\hat{j}$

$\vec{u}_x = 6\hat{i}$

$u_y = 8\,\hat{j}$

$R = \dfrac{2u_x u_y}{g} = \dfrac{2\times 6\times 8}{10} = 9.6$

Q.8 (4)

$R = u^2 \sin 2\theta / g$

$R_{max} = u^2/g$

$22 = \dfrac{u^2}{g}$

for $\theta = 15°$

$R = \dfrac{u^2 \sin 30}{g} = 22\times\dfrac{1}{2} = 11\text{m}$

Q.9 (4)

$y = x\tan\theta\left(1 - \dfrac{x}{R}\right)$

$y = x\tan\theta - \tan\theta\dfrac{x^2}{R}$

Compare from eq^n.

$\tan\theta = 16$

$\dfrac{\tan\theta}{R} = \dfrac{5}{4}$

$R = \dfrac{64}{5} = 12.8\,\text{m}$

$\tan\theta = 16$

$\dfrac{1}{2}\dfrac{g}{u^2\cos^2\theta} = \dfrac{5}{4.2}$

$R = \dfrac{u^2\sin 2\theta}{g} = \dfrac{2g}{5}\times\dfrac{2\times 16}{g}$

$R = 12.8\text{m}$

Q.10 (3)

Range of θ and 90-θ is same

If $\theta = 30°$

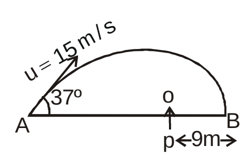
So $90 - \theta = 60°$

Q.11 (3)

For both particles $u_y = 0$ and $a_y = -g$

$h = \dfrac{1}{2}gt^2 \Rightarrow h \rightarrow \text{same} \Rightarrow t \rightarrow \text{same}$

Q.12 (2)



In this process both time taken is same.

$T = \dfrac{2u\sin\theta}{g}$

$T = \dfrac{2u\sin 37°}{g}$

$= \dfrac{2\times 15\times 3}{10\times 5} = 1.8\,\text{Sec}$

Minimum Velocity $= \dfrac{9}{1.8} = 5\,\text{m/s}$

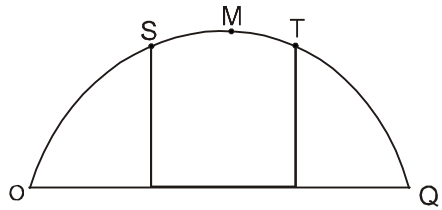
Q.13 (2)

$\therefore H_{max} = \dfrac{u_y^{\,2}}{2g}$

$\Rightarrow u_y = \sqrt{2gH}$

$\therefore T = \frac{2u_y}{g} = \frac{2\sqrt{2gH}}{g} = 2\sqrt{2}\sqrt{\frac{H}{g}}$

Q.14 (3)



$t_{(OS)} = 1$ sec

$t_{(OT)} = 3$

or $t_{(ST)} = t_{(OT)} - t_{(OS)} = 3 - 1 = 2$ sec

$\therefore t_{(SM)} = \frac{1}{2} t_{(ST)} = 1$ sec.

$\therefore t_{(OM)} = t_{(OS)} + t_{(SM)} = 1 + 1 = 2$sec.

$\therefore$ Time of flight= $2 \times 2 = 4$ sec. **Ans. "C"**

Q.15 (2)

As $\theta\uparrow$, H and T both increases

But R $\uparrow$ from 0º to 45º & at $\theta = 45°$ Max then decreases

Ans (2) R$\uparrow$ then $\downarrow$ [θ from from 30º to 60º]

while H$\uparrow$ and T$\uparrow$.

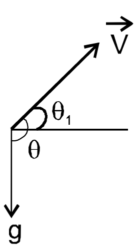
Q.16 (4)

Acute Angle of Velocity with horizontal possible is – 90º to + 90º hence angle with g is 0° to 180º.

θ_1 is acute

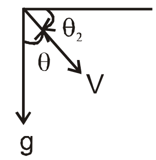
$\Rightarrow 0° \le \theta_1 < 90°$(during the upward journey of mass)

from fig. $\theta = 90° + \theta_1$



or, $90° \le \theta < 180°$ ....(1)

During downward motion



$0° < \theta_2 < 90°$

$\theta = 90° - \theta_2$

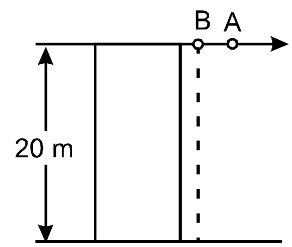
$0° < \theta < 90°$ ....(2)

From eq. (1) and (2)

i.e., $0 < \theta < 90°$ U $90° \le \theta < 180°$

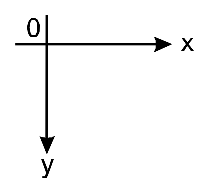
$\Rightarrow 0° < \theta < 180°$ "D" **Ans.**

Q.17 (4)



10 m/s = u_A ;

$u_B = 0$ m/s $u_A = 10\,\hat{i}$



On reaching the ground,

Both will have same vertical velocity

$V_y^2 = u_y^2 + 2a_y s_y$

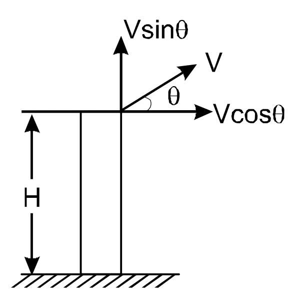
since $u_y = 0$ for both A & B

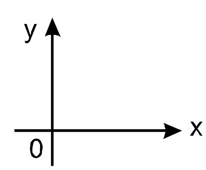
$a_y = g$ for both A & B

$s_y = 20$ m for both A & B

Thats why the time taken by both are same

Q.18 (4)





Let final vel be V_2

Now v_{2x} = horizontal component of velocity

$V_{2x} = V\cos\theta$ & $V^2_{2y} = (V\sin\theta)^2 + 2(-g)(-H)$

$\therefore\ V^2_{2y} = V^2\sin^2\theta + 2gH$

$\Rightarrow\ V^2_2 = V^2_{2x} + V^2_{2y}$

$= (V\cos\theta)^2 + [V^2\sin^2\theta + 2gH]$

$V^2_2 = V^2 + 2gH$

i.e., $V = \sqrt{v^2 + 2gH}$ $\left\{V_2 = \sqrt{V^2 + 2gH}\right\}$

This magnitude of final velocity is independent on θ

$\Rightarrow$ all particles strike the ground with the same speed.

i.e., 'A' is correct.

In vertical motion

The highest velocity (initial) along the direction of displacement is possessed by particle (1). Hence particle (1) will reach the ground earliest. [since a_y and s_y are same for all]

i.e., 'C' is correct

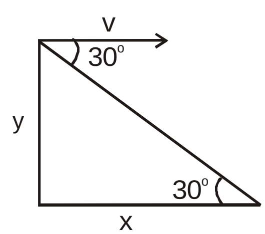
Q.19 (1)

$AC = \frac{1}{2}gt^2 = 45$ m $BC = 45\sqrt{3}$ m = u.t

$u = \frac{45}{\sqrt{3}} = 15\sqrt{3}$ m/s.

Alter : Object is thrown horizontally so $u_x = v$ & $u_y = 0$

from Diagram



$-y = u_y t - \frac{1}{2}gt^2$

$y = \frac{1}{2}\times10\times(3)^2$

$y = 45$m ....(1)

& $\tan 30^o = \frac{y}{x} \Rightarrow y\sqrt{3} = x$ ....(2)

& $x = vt = 3v$ ....(3)

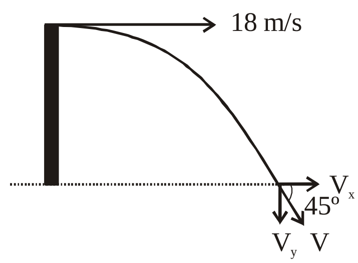
from equation (1), (2) & (3)

$45\sqrt{3} = 3v$

$v = 15\sqrt{3}$ m/s

Q.20 (2)

$\tan 45° = \frac{v_y}{v_x}$



$\Rightarrow v_y = v_x = 18$m/s **Ans.**

Q.21 (4)

$(Y_{max}) \Rightarrow \frac{dY}{dt} = 0$

$\Rightarrow \frac{d}{dt}(10t - t^2) = 10 - 2t \Rightarrow t = 5$

$\Rightarrow Y_{max} = 10(5) - 5^2 = 25$ m **Ans** "D"

Q.22 (4)

(i)

For θ and $90-\theta$

Range is ssame

$\theta = 15^0$

$90-\theta = 75^0$

(ii) $R = \frac{u\sin\theta . u\cos\theta}{g}$

$\therefore \sin(90-\theta) = \cos\theta$

$\therefore \cos(90-\theta) = \sin\theta$

Q.23 (2)

$E = \frac{1}{2}mv^2$

At Highest Point

vel = $v\cos\theta$

$KE = \frac{1}{2}mv^2\cos^2\theta = E\cos^2\theta = \frac{E}{2}$ $(\therefore \theta = 45°)$

Q.24 (2)

Vel. of Bomb is same as the vel. of aeroplane.

$u_x = 360$ km/h & $v_y = 0$.

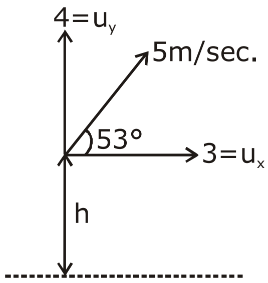
$S_y = u_y t + \frac{1}{2}a_y t^2$,   Here u = 0

$1960 = \frac{1}{2}\times 9.8t^2$

t = 20 sec

Q.25 (3)

$v_y^2 - u_y^2 = 2a_yS$



$v_y^2 - (4)^2 = -2\times10\times0.45$

$v_y^2 = 7m^2/s^2$

$v_x = 5\cos53^o = 3m/s$ (always remains same)

$\therefore V_{net} = \sqrt{(V_x)^2 + (V_y)^2} = \sqrt{9+7}$

$= 4m/s$

Q.26 (2)

$S = \frac{1}{2}at^2$

$h = \frac{1}{2}gt_1^2 \Rightarrow t_1 = \frac{\sqrt{2h}}{g}$

$x = v_1t_1$

$x = v\sqrt{\frac{2h}{g}}$ ....(i)

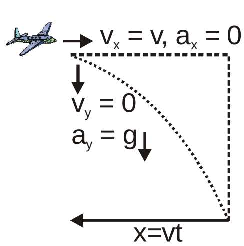
$t_2 = \frac{\sqrt{4h}}{g}$, $2x = v_2 \times t_2$

$2x = v_2\sqrt{\frac{4h}{g}}$ ....(ii)

From (i) & (ii)

$V' = \sqrt{2}V$

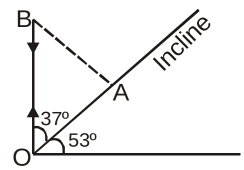
Q.27 (1)



Because horizontal velocity of plane and bomb is always same.

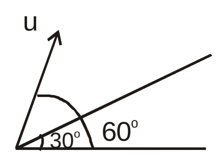
Q.28 (1)

$OB = \frac{u^2}{2g} = 5m$



$\therefore$ $AB = OB\sin37^o = 3m.$

Q.29 (3)



$u = 10\sqrt{3}$ m/s

Time of flight on the incline plane

$T = \frac{2u\sin\alpha}{g\cos\beta}$

given $\alpha = 30^o$ & $\beta = 30^o$ & $u = 10\sqrt{3}$ m/s

$T = \frac{2\times10\sqrt{3}\sin30^o}{10\cos30^o}$

so T= 2 sec .

Q.30 (2)

At maximum height $v = u\cos\theta$

$\frac{u}{2} = v$

$\Rightarrow \frac{u}{2} = u\cos\theta$

$\Rightarrow \cos\theta = \frac{1}{2}$

$\Rightarrow \theta = 60°$

$R = \frac{u^2\sin2\theta}{g} = \frac{u^2\sin(120°)}{g}$

$= \frac{u^2\cos30°}{g} = \frac{\sqrt{3}u^2}{2g}$

JEE-ADVANCED
OBJECTIVE QUESTIONS

Q.1 (A)

$a_x = 2\ m/s^2$; $a_y = 0$
$u_x = 8$ m/s
$u_y = -15$ m/s.

$\vec{V} = V_x\,\hat{i} + V_y\,\hat{j}$

$V_y = u_y + a_y t$

$\Rightarrow V_y = -15$ m/s

$V_x = u_x + a_x t$

$V_x = 8 + 2t$

$\Rightarrow V = [(8 + 2t)\,\hat{i} - 15\,\hat{j}]$ m/s.

Q.2 (D)

$u = 3m/s\,\hat{i}$ $a = 1\,\hat{j}\,m/s^2$

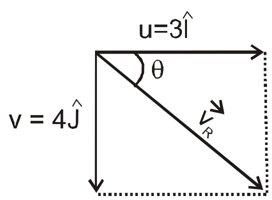
a is $\perp$ u

so V after 4 sec

V = u + at

$V = 3\hat{i} + 1\hat{j} \times 4$

$V = 3\hat{i} + 4\hat{j}$



$\vec{V}_R = \sqrt{u^2 + v^2} = \sqrt{3^2 + 4^2}$

$\vec{V}_R = 5$ m/s

and $\tan\theta = \dfrac{4}{3}$

$\theta = \tan^{-1}\left(\dfrac{4}{3}\right)$ with the direction of the initial velocity.

Q.3 (B)

$x = 5\sin 10t \Rightarrow v_x = \dfrac{dx}{dt} = 50\cos 10t$

$y = 5\cos 10t \Rightarrow v_y = \dfrac{dy}{dt} = -50\sin 10t$

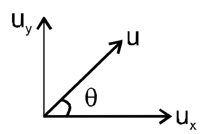
$V^2_{net} = V_x^{\,2} + v_y^{\,2}$

$v_{net} = \sqrt{(50)^2(\sin^2 10t + \cos^2 10t)} = 50$ m/sec

Q.4 (D)

V = u + at

V_y = reduced then increases



$\Rightarrow$ V = reduced then increase ($\because V_x$ is constant)

$\Rightarrow$ Speed first reduces than increases. So "A" is not correct

$KE = \dfrac{1}{2}mV^2 = \dfrac{m}{2}(\text{speed})^2$

$\Rightarrow$ "B" is not correct

V_y = changes

$\Rightarrow$ "C" is not correct.

V_x = constt. since grairty is vertically down

$\Rightarrow$ no component of acceleration along the horizontal direction.

$\Rightarrow$ "D" is correct

Q.5 (B)

$y = ax^2$ ....(1)

given $V_x = C$

from (1) $\dfrac{dy}{dt} = 2ax.\dfrac{dx}{dt}$

$V_y = 2ax.C$ ....(2)

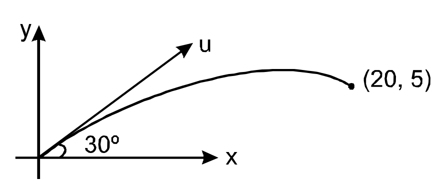
from (2) $\dfrac{dv_y}{dt} = 2ac.\dfrac{dx}{dt}$

$a_y = 2acV_x$

$a_y = 2ac^2$

$\vec{a}_y = 2ac^2\,\hat{j}$

Q.6 (A)



$y = x\tan\theta - \dfrac{1}{2}g\dfrac{x^2}{u^2\cos^2\theta}$

$$5 = 20 \tan 30^o - \frac{1}{2} \times \frac{10 \times 20^2}{u^2 \cos^2 30^o}$$

$$\Rightarrow u^2 = \frac{1600}{\sqrt{3}(4-\sqrt{3})} = \frac{1600}{13\sqrt{3}} (4+\sqrt{3})$$

Q.7 (A)

$$\frac{R^2}{8h} + 2h = \frac{\left(\frac{u^2 2\sin\theta\cos\theta}{g}\right)^2}{\frac{8 \times u^2 \sin^2\theta}{2g}} + 2\frac{u^2 \sin^2\theta}{2g}$$

$$= \frac{u^2}{g} \text{ (max . horizontal Range)}$$

Q.8 (A)

$x = y^2 + 2y + 2$

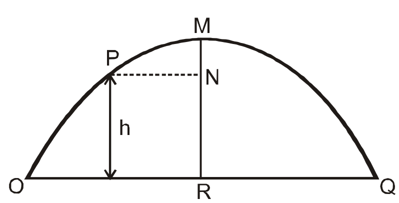
$$\frac{dx}{dt} = 2y\frac{dy}{dt} + 2\frac{dy}{dt} + 0$$

$$\frac{d^2x}{dt^2} = 2\left(\frac{dy}{dt}\right)^2 + 2y\frac{d^2y}{dt^2} + 2\frac{d^2y}{dt^2}$$

$$\frac{d^2y}{dt^2} = 0 \,.\, \left(\frac{dy}{dt} = \quad 5 \text{ m/s}\right)$$

$$\frac{d^2x}{dt^2} = 2\,(5^2) + 0 + 0 = 50 \text{ m/s}^2$$

Q.9 (A)



$t_{op} = 4$
$t_{PQ} = 5$
$\therefore t_{OQ} = 4 + 5 = 9$

or $\tan = \frac{9}{2} = 4.5$

$$RM - MN = h = \frac{1}{2} g\,[(4.5)^2 - (.5)^2]$$

$$= \frac{1}{2} \; 98 \;.5.1 = 98$$

Q.10 (D)

From the R v/s θ curve (for u = const.)

$$R_{max} = \frac{u^2}{g} = 250 \Rightarrow u = 50 \text{ m/sec.}$$

$T = 1/2\ T_{max.}$ possible

$$\frac{2u\sin\theta}{g} = \frac{1}{2}\left(\frac{2u}{g}\right)$$

$$\Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$$

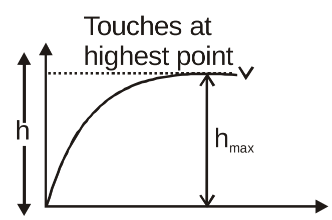
Least speed during flight = u cos θ = 50 cos30 = $25\sqrt{3}$

Q.11 (B)

$$\vec{V} = a\hat{i} + (b - ct)\hat{j} = u_x\hat{i} + (u_y - gt)\hat{j}$$

$$R = \frac{2u_x u_y}{g} = \frac{2ab}{c}$$

Q.12 (C)

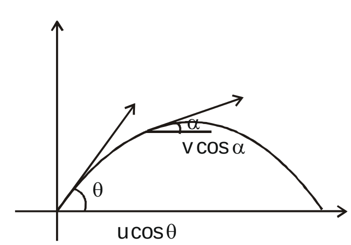


$$h = \frac{U_y^2}{2g}$$

$$U_y = \sqrt{2gh}$$

$$R = U_x T = \frac{U.\sqrt{2gh}}{g} \quad \Rightarrow \quad R = U\sqrt{\frac{2h}{g}}$$

Q.13 (C)



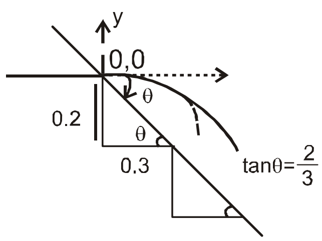
Because horizontal component of the vel. is never

change in projectile motion.
Horizontal Component

$u\cos\theta = v\cos\alpha$

$\therefore\ v = u\cos\theta\sec\alpha$

Q.14 (A)



we have the point of projection as (0, 0)
we have the equation of st. line (as shown in fig.)

$y = \tan\theta / x$

$$y = \frac{-2}{3} \times \qquad \text{....(1)}$$

Also the equation of trajectory for horizantal projetion

$$y = \frac{-1}{2}\, g\, \frac{x^2}{u^2} \qquad \text{....(2)}$$

from (1) and (2)

$$\frac{1}{2}\, g\, \frac{x^2}{u^2} = \frac{2}{3}\, x$$

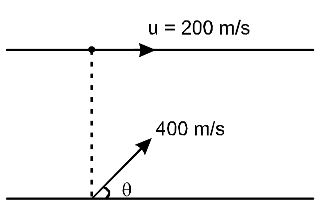
$$\text{or}\quad X = \frac{2}{3} \times \frac{u^2}{5}$$

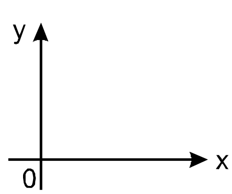
$$= \frac{2}{3} \times \frac{4.5 \times 4.5}{5} = 3 \times 0.9$$

If no. of stps be n then $n \times 0.3 = 3 \times 0.9$

$n = 9$

Q.15 (B)





To hit, $400\cos\theta = 200$
{$\because$ Both travel equal distance along horizontal, of their start and coordinates of x axis are same}
$\Rightarrow\ \theta = 60^\circ$ **Ans.**

Q.16 (C)
Let initial and final speeds of stone be u and v.

$\therefore\ v^2 = u^2 - 2gh$ ....(1)

and $v\cos 30^\circ = u\cos 60^\circ$ ....(2)

solving 1 and 2 we get

$u = \sqrt{3gh}$

Q.17 (D)
Since time of flight depends only on vertical component of velocity and acceleration . Hence time of flight is

$$T = \frac{2u_y}{g} \quad \text{where } u_x = \cos\theta \text{ and } u_y = u\sin\theta$$

$\therefore$ In horizontal (x) direction

$$d = u_x t + ½\, gt^2 = u\cos\theta \left(\frac{2u\sin\theta}{g}\right) + \frac{1}{2} g \left(\frac{2u\sin\theta}{g}\right)^2$$

$$= \frac{2u^2}{g}\ (\sin\theta\cos\theta + \sin^2\theta)$$

We want to maximise $f(\theta) = \cos\theta\sin\theta + \sin^2\theta$

$\Rightarrow\ f'\theta = -\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 0$

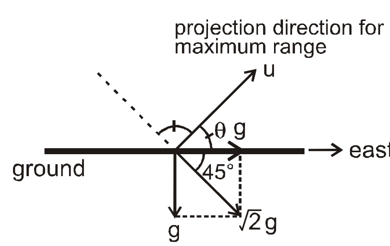
$\Rightarrow\ \cos 2\theta + \sin 2\theta = 0$

$\Rightarrow\ \tan 2\theta = -1$

$$\text{or } 2\theta = \frac{3\pi}{4} \text{ or } \theta = \frac{3\pi}{8} = 67.5^\circ$$

Alternate :
As shown in figure, the net acceleration of projectile makes on angle 45° with horizontal. For maximum range on horizontal plane, the angle of projection should be along angle bisector of horizontal and opposite direction of net acceleration of projectile.

$$\therefore\ \theta = \frac{135^\circ}{2} = 67.5^\circ$$



Q.18 (A)

From given conditions $V_A = V_B \cos 37^0 = 15.\frac{4}{5} = 12$ m/sec.

$\therefore$ time of flight of A (t) = $\sqrt{\frac{2\times 20}{10}}$ = 2 sec.

$\Rightarrow$ Range $= V_A t = 24$ m

Q.19 (C)

H=20 m, $u_y = 0$

$+S_y = u_y t + \frac{1}{2} g t^2 \therefore \quad t = \sqrt{\frac{2 s_y}{g}}$

$T = \sqrt{\frac{2\times 20}{10}} = 2\,\text{sec}$

Range $= u_x t + \frac{1}{2} a_x t^2$

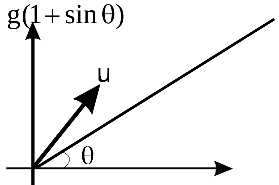
Here $u_x = 0$, $a_x = 6\text{m/s}^2$ (due to wind)

$= 0 + \frac{1}{2}\times 6\times (2)^2 = 12\text{m}$

Q.20 (B)

On the incline plane the maximum possible Range is –

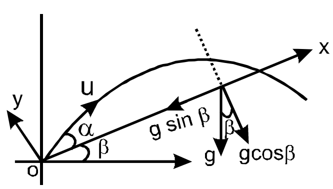
$R = \frac{V^2}{g(1+\sin\theta)}$



Range max = ?

Let $\theta = \beta$

And angle of projection from the inclined plane = α



$u_y = u \sin\alpha$ $u_x = \cos\alpha$ $a_x = -g \sin\beta$ $a_y = -g \cos\beta$

Range $= s_x = u_x T + \frac{1}{2} a_x T^2$

(on the inclined plane) where $T = \frac{2u_y}{g_y}$

$\Rightarrow s_x = (u \cos\alpha)$

$$\left[\frac{2u\sin\alpha}{g\cos\beta}\right] + \frac{1}{2}[-g\sin\beta]\left[\frac{2u\sin\alpha}{2\cos\beta}\right]^2$$

$$= \frac{2u^2\sin\alpha}{g\cos^2\beta}[\cos\alpha\cos\beta - \sin\alpha\sin\beta]$$

$$s_x = \frac{2u^2\sin\alpha}{g\cos^2\beta}[\cos(\alpha+\beta)]$$

$$= \frac{u^2}{g\cos^2\beta}[2\sin\alpha\cos(\alpha+\beta)]$$

$$= \frac{u^2}{g\cos^2\beta}[\sin(2\alpha+\beta)+\sin(-\beta)]$$

$$s_x = \frac{u^2}{g\cos^2\beta}[\sin(2\alpha+\beta)-\sin\beta]$$

Now s_x is max s_x

when $\sin(2\alpha+\beta)$ is max ($\because \beta =$ constt.)

$$\Rightarrow 2\alpha+\beta = \frac{\pi}{2} \Rightarrow \alpha = \frac{\left(\frac{\pi}{2}-\beta\right)}{2}$$

i.e., when ball is projected at the angle bisector of angle formed by inclined plane and dir. of net accelaration reversed.

$$\& (s_x)_{max} = \frac{u^2}{g\cos^2\beta}[1-\sin\beta] = \frac{u^2(1-\sin\beta)}{g(1-\sin\beta)(1-\sin\beta)}$$

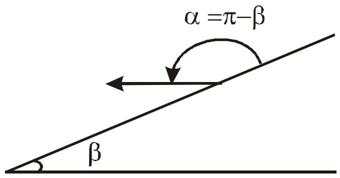
Max. Range on an inclined plane = $\frac{u^2}{g(1+\sin\beta)}$

Here $\beta = \theta \Rightarrow R_{max} = \frac{u^2}{g(1+\sin\theta)}$ **Ans** "B"

Q.21 (D)

$$R = \frac{v^2}{g\cos^2\beta}[\sin(2\alpha+\beta)-\sin\beta]$$

Putting $\beta = 45^o$ & $\alpha = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$



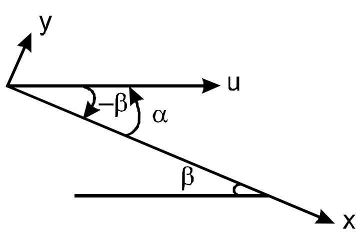
$$R = \frac{v^2}{g\left(\frac{1}{\sqrt{2}}\right)^2}\left[\sin\left(2\times\frac{3\pi}{24}+\frac{\pi}{4}\right)-\sin\frac{\pi}{4}\right]$$

$$s_x = \frac{v^2\times 2}{g}\left[-2\times\frac{1}{\sqrt{2}}\right] = -2\sqrt{2}\,\frac{v^2}{g}$$

(–ve sign indicates that the displacement is in –ve x direction)

$\Rightarrow$ Range = $2\sqrt{2}\,\frac{v^2}{g}$ **Ans** "D"

Alternate II method



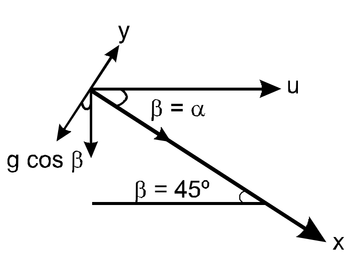
$$\beta = -\frac{\pi}{4} \ \& \ \alpha = \frac{\pi}{4}$$

$$R = \frac{u^2}{g\cos^2\beta}\left[\sin(2\alpha+\beta)-\sin\beta\right]$$

$$= \frac{u^2}{g\left(\frac{1}{\sqrt{2}}\right)^2}\left[\sin\frac{\pi}{4}-\sin\left(-\frac{\pi}{4}\right)\right]$$

$$R = \frac{2\sqrt{2}\ u^2}{g}$$ (along +ve x die.) (+ve x)

III Method



$u_x = u\cos\beta$, $T = \left|\frac{2u\sin\beta}{g\cos\beta}\right|$, $a_x = g\sin\beta$;

$a_y = -\cos\beta$

$s_x = u_x t + \frac{1}{2} a_x t^2 = (u\cos\beta)$

$$\left[\frac{2u\sin\beta}{g\cos\beta}\right]+\frac{1}{2}(g\sin\beta)\left(\frac{2u\sin\beta}{g\cos\beta}\right)^2$$

Let $\alpha = \beta = 45°$

So, $s_x = \frac{u^2 2}{g}\left(\frac{1}{\sqrt{2}}\right)+\frac{1}{2}g\left(\frac{1}{\sqrt{2}}\right)\frac{2.2u^2}{g^2} = \frac{2}{\sqrt{2}}\frac{u^2}{g}[1+1]$

$s_x = 2\sqrt{2}\,\frac{u^2}{g}$

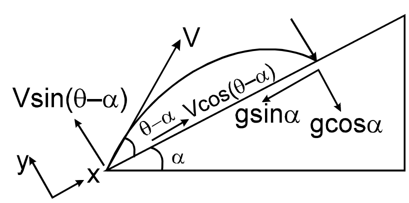
Ans "D"

Q.22 (D)

Applying equation of motion perpendicular to the incline for y = 0.

$$0 = V\sin(\theta-\alpha)t + \frac{1}{2}(-g\cos\alpha)\,t^2$$

$$\Rightarrow\ t = 0 \ \& \ \frac{2V\sin(\theta-\alpha)}{g\cos\alpha}$$



At the moment of striking the plane, as velocity is perpendicular to the inclined plane hence component of velocity along incline must be zero.

$$0 = v\cos(\theta-\alpha) + (-g\sin\alpha).\ \frac{2V\sin(\theta-\alpha)}{g\cos\alpha}$$

$v\cos(\theta-\alpha) = \tan\alpha.\ 2V\sin(\theta-\alpha)$

$\cot(\theta-\alpha) = 2\tan\alpha$ **Ans. (D)**

Q.23 (C)

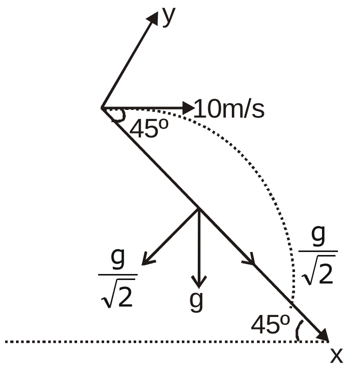
$a_y = -\,{}^{g}\!/\!_{\sqrt{2}}$ m/s^2

$a_x = {}^{g}\!/\!_{\sqrt{2}}$ m/s^2

$$T = \frac{2u\sin\theta}{g_{eff}}$$

$$T = \frac{2\times(10/\sqrt{2})}{g/\sqrt{2}}$$

$= 2$ sec

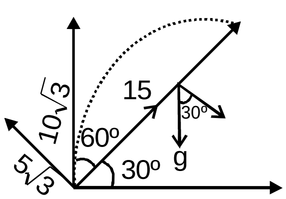


Q.24 (C)

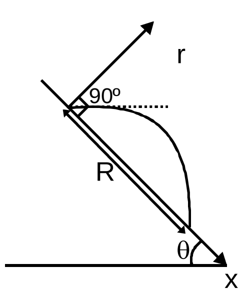
$$T = \frac{2u_y}{g\cos\theta}$$

$$T = \frac{2\times 5\sqrt{3}}{10\times\cos 30^o}$$

$T = 2$sec



Q.25 (C)



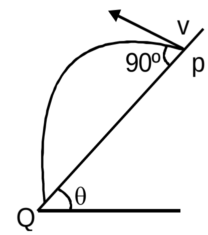
$a_y = -g\cos\theta$
$a_x = g\sin\theta$
$u_y = v$
$u_x = 0$

$$\text{Range} = \frac{1}{2}a_x T^2$$

$$T = \frac{2.v}{g\cos\theta} = \frac{1}{2}g\sin\theta\left(\frac{2v}{g\cos\theta}\right)^2$$

$$R = 2\frac{v^2}{g}\tan\theta\sec\theta$$

Q.26 (D)



$$R = u_x t + \frac{1}{2}a_x t^2$$

$$[\therefore u_x = 0.\ u_y = v]$$

$$\Rightarrow \therefore\ T = \frac{2u_y}{g\cos\theta} = \frac{2v}{g\cos\theta}$$

$$R = \frac{1}{2}\ g\sin\theta\ T^2$$

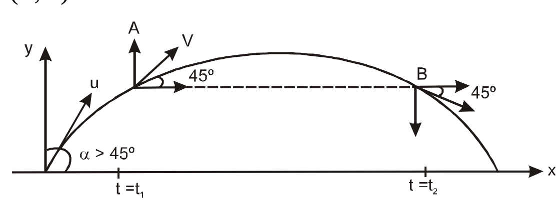
$$R = \frac{1}{2}\ g\sin\theta\left(\frac{2V}{g\cos\theta}\right)^2$$

$R = Tv\tan\theta$

JEE-ADVANCED

MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 (B, C)



$\Rightarrow$ Time at which $V_x = V_y$ is what we are solving $V_x = V_y$

Now, $V_x = u\cos\alpha$

$V_y = u\sin\alpha - gt$

$\Rightarrow$ $u\cos\alpha = u\sin\alpha - gt$ {$\because$ $V_y = V_x$} ; at $t = t_1$ (say)

$\Rightarrow$ $t_1 = \frac{u}{g}$ $(\sin\alpha - \cos\alpha)$ **"C" Ans**

Also when $V_y \equiv V_x$

{i.e., when we choose 'y' axis as – y'} at $t = t_2$ (say)

$-u\cos\alpha = u\sin\alpha - gt_2$

$\Rightarrow\ t_2 = \frac{u}{g}\ (\sin\alpha + \cos\alpha)$ **"B" Ans**

Q.2 (A, B)

$x = 24 = u\cos\theta.t$

$\Rightarrow\ t = \frac{24}{24\cos\theta} = \frac{1}{\cos\theta}$

$y = 14 = u\sin\theta t - \frac{1}{2}gt^2$

$\Rightarrow\ 14 = \frac{u\sin\theta}{\cos\theta} - \frac{5}{\cos^2\theta}$

$\Rightarrow\ 14 = u\tan\theta - 5\sec^2\theta$

$\Rightarrow\ 5\tan^2\theta - 24\tan\theta + 19 = 0$

$\Rightarrow\ \tan\theta = 1, 19/5$. **Ans**

Q.3 (C)

$R = \frac{u^2\sin 2\theta}{g}$

$\sqrt{3} = \frac{20\sin^2\theta}{10} \Rightarrow \sin 2\theta = \frac{\sqrt{3}}{2}$

$\therefore\ \theta = 30^o$

(A) $H_{max} = \frac{u^2\sin 2\theta}{2g} = \frac{(\sqrt{20})^2 \times \frac{1}{4}}{2\times 10} = 0.25m$

(B) Minimum Velocity $u\cos\theta$

$= \sqrt{20}\times\cos 30^o$

$= \sqrt{20}\times\frac{\sqrt{3}}{2} = \sqrt{15}m/s$

(C) $T = \frac{2u\sin\theta}{g}$

$= \frac{2\times\sqrt{20}\times\frac{1}{2}}{10}$

$\Rightarrow\ \sqrt{\frac{1}{5}}$ sec

Q.4 (A,B,C,D)

$h = \frac{u^2}{2g} \Rightarrow u = \sqrt{2gh}$

(a) $R_{max} = \frac{u^2}{g} = 2h$

(b) $R = nH_{max}$

$\frac{u^2\sin 2\theta}{g} = n\frac{u^2\sin^2\theta}{2g}$

$4 = n\tan\theta$

$\theta = \tan^{-1}\left(\frac{4}{n}\right)$

(c) $gT^2 = g\times\frac{4u^2\sin^2\theta}{g^2} = \frac{2\times u^2\sin 20}{g}\times\tan\theta$

$gT^2 = 2R\tan\theta$

(d) $T = \frac{2u_y}{g}, H_{max} = \frac{u_y^{\ 2}}{2g}$

$\therefore$ Ratio 1:1

Q.5 (A,B)

Put the value of T, R, H, in the given equation and solve each option.

Q.6 (A,C,D)

$T = \sqrt{\frac{2H}{g}} \Rightarrow 0.4 = \sqrt{\frac{2H}{g}}$

$\Rightarrow H = 0.8m$

$R = 0.4\times 4 = 1.6m$

and $U_y = \sqrt{2gH} = \sqrt{2\times 10\times 0.8} = 4m/s \Rightarrow\ \theta = 45^o$

Comprehension-1 (Q. No 7 to 9)

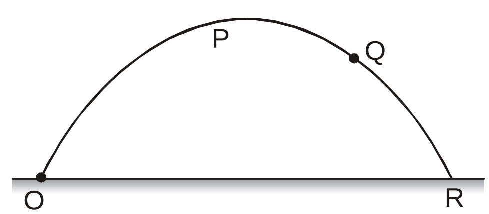
Q.7 **(A)**
Q.8 **(C)**
Q.9 **(C)**

Sol.7 $H_A = H_C > H_B$

Obviously A just reaches its maximum height and C has crossed its maximum height which is equal to A as u and θ are same. But B is unable to reach its max. height.

Sol.8 Time of flight of A is 4 seconds which is same as the time of flight if wall was not there.

Time taken by C to reach the inclined roof is 1 sec.



$T_{OR} = 4$

$T_{QR} = 1$

$\therefore\ T_{OQ} = T_{OR} - T_{QR} = 3$ seconds

Sol.9 from above $= \frac{2u\sin\theta}{g} = 4$

$\therefore$ $u\sin\theta = 20$ m/s $\Rightarrow$ vertical component is 20 m/s.

for maximum height

$v^2 = u^2 + 2as \Rightarrow 0^2 = 20^2 - 2 \times 10 \times s$

$s = 20$ m.

Comprehension-2 (Q. No 10 to 13)

Q.10 (B)

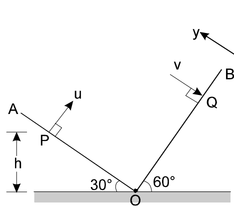
Q.11 (A)

Q.12 (C)

Q.13 (A)

Sol. Let us choose the x and y directions along OB and OA respectively. Then

$u_x = u = 10\sqrt{3}$ m/s, $u_y = 0$



$a_x = -g\sin 60° = -5\sqrt{3}$ m/s^2

and $a_y = -g\cos 60° = -5$ m/s^2

Sol.10 At point Q, x-component of velocity is zero. Hence, substituting in

$v_x = u_x + a_x t$

$0 = 10\sqrt{3} - 5\sqrt{3}t$

or $t = \frac{10\sqrt{3}}{5\sqrt{3}} = 2s$ **Ans.**

Sol.11 At point Q, $v = v_y = u_y + a_y t$

$\therefore$ $v = 0 - (5)(2) = -10$ m/s **Ans.**

Here, negative sign implies that velocity of particle at Q is along negative y direction.

Sol.12 Distance PO = |displacement of particle along y-direction |

Here, $s_y = u_y t + \frac{1}{2} a_y t^2$

$= 0 - \frac{1}{2}(5)(2)^2 = -10$ m

$\therefore$ PO = 10 m

Therefore, $h = PO \sin 30° = (10)\left(\frac{1}{2}\right)$

or h = 5 m Ans.

Sol.13 Distance OQ = displacement of particle along x-direction = s_x

Here $s_x = u_x t + \frac{1}{2} a_x t^2$

$= (10\sqrt{3})(2) - \frac{1}{2}(5\sqrt{3})(2)^2 = 10\sqrt{3}$ m

or $OQ = 10\sqrt{3}$ m

$\therefore$ $PQ = \sqrt{(PO)^2 + (OQ)^2} = \sqrt{(10)^2 + (10\sqrt{3})^2} =$

$\sqrt{100+300} = \sqrt{400}$

$\therefore$ PQ = 20 m **Ans.**

Comprehension-3 (Q. No 14 to 17)

Q.14 (B)

Q.15 (C)

Q.16 (D)

Q.17 (B)

Sol. 14 $a_{AB} = 0$

$\therefore$ Straight line

Sol.15 Given $V_1\cos\theta_1 = v_2\cos\theta_2 \Rightarrow v_{xA} = V_{xB}$

$\vec{v}_A = V_{XA}\hat{i} + V_{yA}\hat{j}$; $\vec{V}_B = V_{XB}\hat{i} + V_{yB}\hat{j}$

$\therefore$ $\vec{v}_{AB} = V_{yA}\hat{j} - V_{yB}\hat{j}$

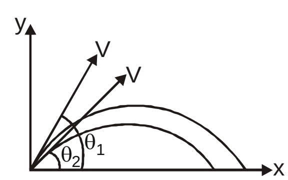
Sol. 16 $T = \frac{2u_y}{g}$

$\therefore$ Same

$H = \frac{u_y^2}{2g}$

$\vec{V}_{AB} = V_{xA}\hat{i} - V_{xB}\hat{i}$

Sol.17



$R = \frac{2u_x u_y}{g} = \frac{u^2\sin 2\theta}{g}$

when θ and $90 - \theta$ range is same

$v_{21} = v_{2x}\hat{i} - v_{2y}\hat{j} - v_{1x}\hat{i} - v_{1y}\hat{j}$

$\tan\theta = \left(\frac{v_{2y} - v_{1y}}{v_{2x} - v_{1x}}\right) \begin{matrix} v_{2y} < v_{1y} \\ v_{2x} > v_{1x} \end{matrix}$

$\tan\theta = -ve$

Q.18 (A) r (B) s (C) q (D) p

Time of flight

$$T = \frac{2u}{g\cos 45°} = \frac{2u}{g\cos 45°} = \frac{2\sqrt{2}u}{g} \quad \therefore D \to p$$

Velocity of stone is parallel to x-axis at half the time of flight.

$\therefore A \to r$

At the instant stone make 45° angle with x-axis its velocity is horizontal.

$$\therefore \text{The time is} = \frac{u\sin 45°}{g} = \frac{u}{\sqrt{2}g} \quad \therefore B \to s$$

The time till its displacement along x-axis is half the range is $\frac{1}{\sqrt{2}}T = \frac{2u}{g} \quad \therefore C \to q$

Q.19 (A) r (B) s (C) q (D) p

Equation of path is given as

$y = ax - bx^2$

Comparing it with standard equation of projectile;

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$\tan\theta = a, \quad \frac{g}{2u^2\cos^2\theta} = b$$

Horizontal component of velocity = $u\cos\theta = \sqrt{\frac{g}{2b}}$

$$\text{Time of flight } T = \frac{2u\sin\theta}{g} = \frac{2(u\cos\theta)\tan\theta}{g}$$

$$= \frac{2\left(\sqrt{\frac{g}{2b}}\right)a}{g} = \sqrt{\frac{2a^2}{bg}}$$

$$\text{Maximum height } H = \frac{u^2\sin^2\theta}{2g} = \frac{[u\cos\theta.\tan\theta]^2}{2g}$$

$$= \frac{\left[\sqrt{\frac{g}{2b}}.\right]^2}{2g} = \frac{a^2}{4b}$$

$$\text{Horizontal range } R = \frac{u^2\sin 2\theta}{2g}$$

$$= \frac{2(u\sin\theta)(u\cos\theta)}{g} = \frac{2\left[\sqrt{\frac{g}{2b}}.a\right]\left[\sqrt{\frac{g}{2b}}\right]}{g} = \frac{a}{b}$$

NUMERICAL VALUE BASED

Q.1 [75 m/s]

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$100 = 450\tan 53° - \frac{10(450)^2}{2u^2\cos^2 53°}$$

u = 75 m/s

Q.2 [5 m/s]

$$\text{Time of flight (T)} = \frac{2u\sin 37°}{g\cos 37°}$$

Relative acceleration along incline is zero

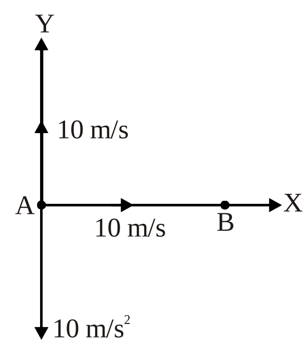
$\therefore$ Relative velocity along incline ucos37° is constant

$$\text{So, } T = \frac{3}{u\cos 37°} = \frac{2u\sin 37°}{g\cos 37°}$$

On solving, we get

u = 5 m/s

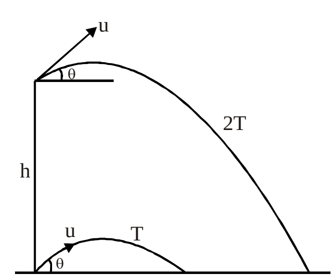
Q.3 [0020]



Path of particle A, w.r.t. B will projectile, they collide if AB in equal to range

$$AB = \frac{2\times10\times10}{10} = 20\text{ m}$$

Q.4 [0040]



$$-h = u\sin\theta\, 2T - \frac{1}{2}g(2T)^2$$

$$T = \frac{2u\sin\theta}{g} \qquad u\sin\theta = \frac{gT}{2}$$

$-h = \frac{gT}{2} \times 2T - \frac{1}{2} g^4T^2$

$-h = -gT^2$

$h = gT^2$ for T = 2 sec.

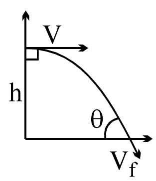
$h = 10 \times 4$ = 40 m

Q.5 [0005]

Minimum θ implies minimum V_{fy} and maximum V_{fx}.

In order to have the aforementioned situation, the rock has to be launched horizontally.

$v_y = \sqrt{2gh}$

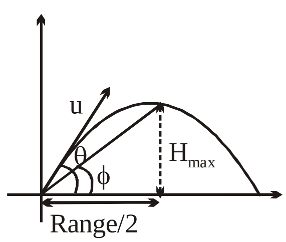


$\tan 30° = \frac{v_y}{v} = \frac{\sqrt{2 \times 10 \times h}}{10\sqrt{3}}$, h = 5 m

Q.6 [0002]

$$\tan\phi = \frac{H_{max}}{\frac{Range}{2}} = \frac{u^2 \sin^2\theta}{2g \times \frac{2u^2 \sin\theta \times \cos\theta}{2g}}$$

$$\tan\phi = \frac{\sin\theta}{2\cos\theta}$$



$$\tan\phi = \frac{\tan\theta}{2}$$

$$\frac{\tan\theta}{\tan\phi} = 2$$

Q.7 [0002]

$$80 = \frac{u^2 \sin 2\theta}{g} \quad (\theta = 45°)$$

$u = 20\sqrt{2}$

$$T = \frac{2u\sin\theta}{g} = \frac{2 \times 20\sqrt{2}}{g} \times \frac{1}{\sqrt{2}}$$

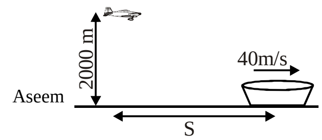
T = 4

$v(T - 0.5) = 7$

$$v = \frac{7}{3.5}$$

v = 2 m/s

Q.8 [1440]



$2000 = \frac{1}{2}gt^2 \quad \Rightarrow \quad t^2 = \frac{4000}{10} \quad \Rightarrow t = 20$ sec

$\therefore S = (112 - 40)(20) = (72)(20) = 1440$ m

Q.9 [0008]

$x = y^2 + 2y + 2$

$$\frac{dx}{dt} = 2x\frac{dy}{dt} + 2\frac{dy}{dt}$$

$v_x = 4y + 4$

$$a_x = \frac{dv_x}{dt} = 4\frac{dy}{dt} = 4 \times 2 = 8 \text{ m/s}^2 \,]$$

Q.10 [10 m]

$R_1 - R_2 = u\cos 45° (t_1 - t_2) = 5\sqrt{2}\,(t_1 - t_2)$

$-y = 5\sqrt{2}\,t_1 - 5t_1^2$ .... (1)

$-y = -5\sqrt{2}\,t_2 - 5t_2^2$ .... (2)

$\Rightarrow \quad (t_1 - t_2) = \sqrt{2}$

$(R_1 - R_2) = 10$

KVPY
PREVIOUS YEAR'S

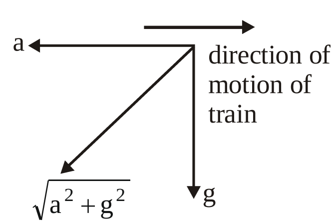
Q.1 (B)

as $V_y = U_y - gt$

Q.2 (A)

Heavier will retard less, it will have more range.

Q.3 (D)



the trajectory of the falling apple as seen by the girl is an incline straight line pointing opposite to the direction of the moving train as relative initial velocity = 0.

Q.4 (D)

Time of fall = $\sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 45}{10}}$

t = 3 sec

horizontal distance = horizontal velocity × time 12 = v × 3

v = 4 m/s

$= 4 \times \frac{18}{5}$ km/hr

v = 14.4 km/hr

v $\approx$ 15 km/hr

Q.5 (B)

Work done by gravity on A & B is same.

$\therefore$ Horizontal velocity of A = horizontal velocity of B as they leave the horizontal portion of the structure.

$\therefore t_A = t_B$ ......(i)

Also vertical velocity of A & vertical velocity of C when released are both zero

$\therefore$ They both will cover same vertical distance in same time.

$\therefore t_A = t_C$ ....(ii)

From (i) & (ii)

$t_A = t_B = t_C$

Note : Correction in option option B should be $t_A = t_B = t_C$

Q.6 (B)

For the same range, another projection angle will be 90° – 30° = 60°

$$h = \frac{u^2 \sin^2 30^o}{2g}$$

$$h_1 = \frac{u^2 \sin^2 60^o}{2g}$$

$$\frac{h_1}{h} = \frac{\sin^2 60^o}{\sin^2 30^o} = 3 \Rightarrow h_1 = 3h$$

Q.7 (B)

Velocity of projection = 24 m/s

Distance between point of projection and hoop

$= \sqrt{25^2 + 45^2}$

$\therefore$ Time taken by ball to reach the hoop

$$= \frac{\sqrt{25^2 + 45^2}}{24}$$

(Note :- We are analysing the motion wrt hoop)

$\therefore$ Distance by which hoop will fall

$$= \frac{1}{2}at^2 = \frac{1}{2}\times 10 \times \frac{\left(25^2 + 45^2\right)}{24^2}$$

$\therefore$ Height above the ground where apple go through the hoop is given by

$$45 - \left[\frac{1}{2}\times 10 \times \frac{\left(25^2 + 45^2\right)}{24^2}\right] = 22\,\text{m}$$

JEE-MAIN PREVIOUS YEAR'S

Q.1 (2)

For $y_{mex} \Rightarrow \frac{dy}{dx} = \alpha - 2\beta x = 0$

$$x = \frac{\alpha}{2\beta}$$

$$y_{max} = H_{max} = \alpha \times \frac{\alpha}{2\beta} - \beta\left(\frac{\alpha}{2\beta}\right)^2$$

$$= \frac{\alpha^2}{2\beta} - \frac{\alpha^2}{4\beta} = \left(\frac{\alpha^2}{4\beta}\right)$$

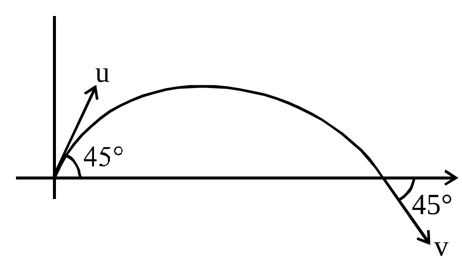
$$= 2x = R = \frac{\alpha}{\beta} = \frac{2u^2 \sin\theta \cos\theta}{g}$$

$$H = \frac{\alpha^2}{4\beta} = \frac{u^2 \sin^2\theta}{2g}$$

$\tan\theta = \alpha$

$\theta = \tan^{-1}(\alpha)$

Q.2



$|\vec{u}| = |\vec{v}|$ ... (1)

$\vec{u} = u\cos 45\hat{i} + u\sin 45\hat{j}$ ... (2)

$\vec{v} = v\cos 45\hat{i} - v\sin 45\hat{j}$ ... (3)

$|\overrightarrow{\Delta P}| = |m(\vec{v} - \vec{u})|$ ... (4)

$\Delta P = 2mu \sin 45°$

$= 2\times5\times10^{-3}\times5\sqrt{2}\times\frac{1}{\sqrt{2}}$

= 50 10–3

= 5 × 10–2

Q.3 [1]

Q.4 [10]

Q.5 (2)

Range $R=\frac{u^2\sin2\theta}{g}$ and same for θ and 90 – θ

So same for 42° and 48°

Maximum height $H=\frac{u^2\sin^2\theta}{2g}$

H is high for higher θ

So H for 48° is higher than H for 42°

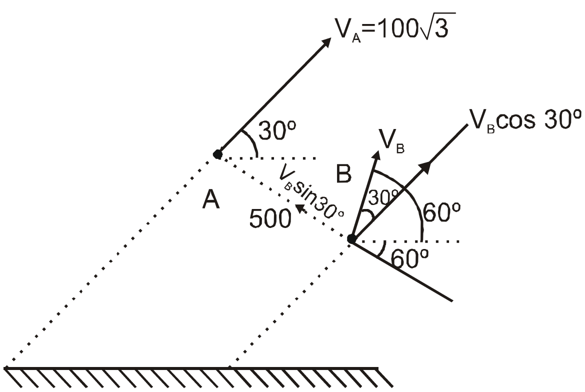
Option (2)

Q.6 (3)

Q.7 (3)

JEE-ADVANCED
PREVIOUS YEAR'S

Q.1 [**5**]



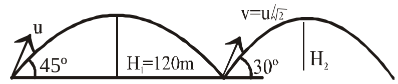
For relative motion perpendicular to line of motion of A

$V_A = 100\sqrt{3} = V_B \cos 30°$

$\Rightarrow V_B = 100$ m/s

$t_0 = \frac{50}{V_B\sin30°} = \frac{500}{200\times\frac{1}{2}} = 5$ sec

Q.2 [30.00]



$H_1=\frac{u^2\sin^2 45}{2g}=120$

$\Rightarrow \frac{u^2}{4g}=120$ ......(i)

when half of kinetic energy is lost $v=\frac{u}{\sqrt{2}}$

$H_2=\frac{\left(\frac{u}{\sqrt{2}}\right)^2\sin^2 30}{2g}=\frac{u^2}{16g}$ ......(ii)

from (i) & (ii)

$H_2=\frac{H_1}{4}=30\text{m}$ on 30.00

Q.3 [4.00]

Average velocity = $\frac{\text{Total displacement}}{\text{Total time}}$

Total time taken = $t_1+t_2+t_3+$ ....

$= t_1+\frac{t_1}{\alpha}+\frac{t_1}{\alpha^2}+$ ....

Total time = $\frac{t_1}{1-\frac{1}{\alpha}}$

Total displacement = $v_1t_1+v_2t_2+$ l

$= v_1t_1+\frac{v_1}{\alpha}.\frac{t_1}{\alpha}+$ ....

$= \frac{v_1t_1}{1-\frac{1}{\alpha^2}}$

On solving

$<v> = \frac{v_1\alpha}{\alpha+1} = 0.8v_1$

$\alpha = 4.00$

Newton's Laws of Motion and Friction

EXERCISES

Elementary

Q.1 (3)

Q.2 (3)

Q.3 (2)

m = 100 m/s, ν = 0, s = 0.06 m

$$\text{Retardation} = \alpha = \frac{u^2}{2s} = \frac{(100)^2}{2\times 0.06} = \frac{1\times 10^6}{12}$$

$$\therefore \quad \text{Force} = ma = \frac{5\times 10^{-3}\times 1\times 10^6}{12} = \frac{5000}{12} = 417 \text{ N}$$

Q.4 (3)

$$\text{Acceleration } \alpha = \frac{F}{m} = \frac{100}{5} = 20 \text{ cm/s}^2$$

$$\text{Now } \nu = \alpha t = 20\times 10 = 200 \text{ cm/s}$$

Q.5 (3)

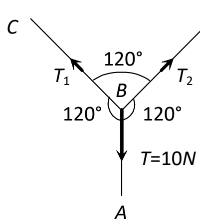
$$\text{Thrust } F = u\left(\frac{dm}{dt}\right) = 5\times 10^4\times 40 = 1\times 10^6 \text{ N}$$

Q.6 (1)

$$F = m\left(\frac{v-u}{t}\right) = \frac{5(65-15)\times 10^{-2}}{0.2} = 12.5 \text{ N.}$$

Q.7 (3)

By drawing the free body diagram of point B



Let the tension in the section BC and BF are T_1 and T_2 respectively.

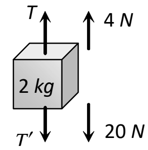
From Lami's theorem

$$\frac{T_1}{\sin 120^\circ} = \frac{T_2}{\sin 120^\circ} = \frac{T}{\sin 120^\circ}$$

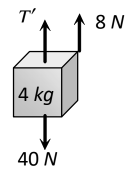
$\Rightarrow T = T_1 = T_2 = 10$ N.

Q.8 (1)

FBD of mass 2 kg FBD of mass 4kg



T – T' – 20 = 4 ….(i)



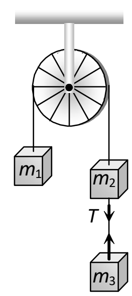
T' – 40 = 8 …(ii)

By solving (i) and (ii) T′ = 47.23 N and T = 70.8 N

Q.9 (1)

Q.10 (2)

Tension between m_2 and m_3 is given by



$$T = \frac{2m_1m_3}{m_1+m_2+m_3}\times g$$

$$= \frac{2\times 2\times 2}{2+2+2}\times 9.8 = 13\text{N.}$$

Q.11 (2)

$$T_2 = (m_A + m_B)\times \frac{T_3}{m_A + m_B + m_C}$$

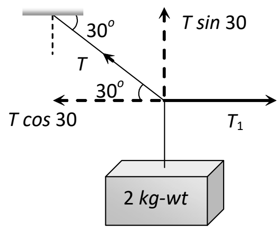
$$T_2 = (1+8)\times \frac{36}{(1+8+27)} = 9 \text{ N.}$$

Q.12 (4)

$$T = \frac{2m_1m_2}{m_1+m_2}g = \frac{2\times 10\times 6}{10+6}\times 9.8 = 73.5\text{N.}$$

Q.13 (3)

T sin 30 = 2kg wt



$\Rightarrow$ T = 4 kg wt

T_1 = T cos 30°

= 4 cos 30°

$= 2\sqrt{3}$.

Q.14 (3)

If monkey move downward with acceleration a then its apparent weight decreases. In that condition

Tension in string = m (g – a)

This should not be exceed over breaking strength of the rope i.e. $360 \ge m(g-a) \Rightarrow 360 \ge 60(10-a)$

$\Rightarrow a \ge 4\,m/s^2$.

Q.15 (3)

As the spring balances are massless therefore the reading of both balance should be equal.

Q.16 (2)

Q.17 (4)

In stationary lift man weighs 40 kg i.e. 400 N.

When lift accelerates upward it's apparent weight = m(g + a) = 40 (10 + 2) = 480 N i.e. 48 kg

For the clarity of concepts in this problem kg-wt can be used in place of kg.

Q.18 (4)

As the apparent weight increase therefore we can say that acceleration of the lift is in upward direction.

R = m (g + a) $\Rightarrow$ 4.8 g = 4 (g + a)

$\Rightarrow$ a = 0.2g = 1.96 m/s^2

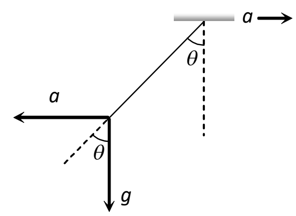
Q.19 (2)

Rate of flow will be more when lift will move in upward direction with some acceleration because the net downward pull will be more and vice-versa.

F_{upward} = m (g + a) and $F_{downward}$ = m (g – a) F_{upward}

Q.20 (1)

When car moves towards right with acceleration a then due to pseudo force the plumb line will tilt in backward direction making an angle θ with vertical.



From the figure,

tan θ = a / g $\qquad\therefore$ θ = tan^{-1} (a/g).

Q.21 (3)

Q.22 (4)

$$\mu = \frac{F}{R} = \frac{F}{mg} = \frac{98}{100\times 9.8} = \frac{1}{10} = 0.1$$

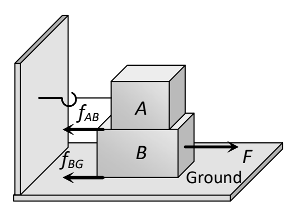
Q.23 (3)

$F_l = \mu_s R$ = 0.4 × mg = 0.4 × 10 = 4N i.e. minimum 4N force is required to start the motion of a body. But applied force is only 3N. So the block will not move.

Q.24 (1)

$$\mu = \frac{m_B}{m_A} \Rightarrow 0.2 = \frac{m_B}{10} \Rightarrow m_B = 2kg$$

Q.25 (3)



$F = f_{AB} + F_{BG}$

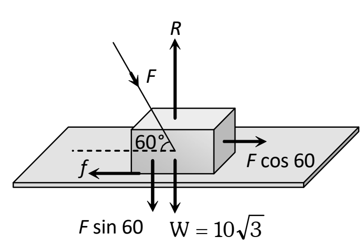
$= \mu_{AB}\, m_A g + \mu_{BG}\,(m_A + m_B)g$

= 0.2 × 100 × 10 + 0.3 (300) × 10

= 200 + 900 = 1100 N

Q.26 (4)

Q.27 (1)



f = μR

F cos 60º = μ(W + F sin 60º)

Substituting $\mu = \frac{1}{2\sqrt{3}}$ & $W = 10\sqrt{3}$ we get F = 20 N

Q.28 (1)

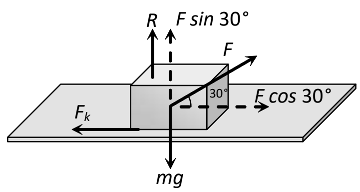
Retarding force F = ma = μR = μ mg ∴ α = μg

Now from equation of motion $v^2 = u^2 - 2\alpha s$

$$\Rightarrow 0 = u^2 - 2as \Rightarrow s = \frac{u^2}{2a} = \frac{u^2}{2\mu g}.$$

Q.29 (4)

Q.30 (1)



Kinetic friction = $\mu_k R$ = 0.2 (mg – F sin 30º)

$$= 0.2\left(5\times10 - 40\times\frac{1}{2}\right) = 0.2(50-20) = 6\text{N}$$

Acceleration of the block

$$= \frac{F\cos 30^o - \text{Kinetic friction}}{\text{Mass}}$$

$$= \frac{40\times\frac{\sqrt{3}}{2} - 6}{5} = 5.73\,\text{m/s}^2.$$

Q.31 (2)

Q.32 (3)

a = g (sin θ – μ cos θ) = 10 (sin 60º – 0.25 cos 60º)

$a = 7.4\ m/s^2$.

Q.33 (1)

Limiting friction between block and slab = $\mu_s m_A g$

= 0.6 × 10 × 9.8 = 58.8 N

But applied force on block A is 100 N. So the block will slip over a slab.

Now kinetic friction works between block and slab FK = $\mu_k m_A g$ = 0.4 × 10 × 9.8 = 39.2 N

This kinetic friction helps to move the slab

$$\therefore \text{ Acceleration of slab} = \frac{39.2}{m_B} = \frac{39.2}{40} = 0.98\,m/s^2.$$

JEE-MAIN OBJECTIVE QUESTIONS

Q.1 (2)

Q.2 (2)

Action and Reaction are equal and opposite

Q.3 (3)

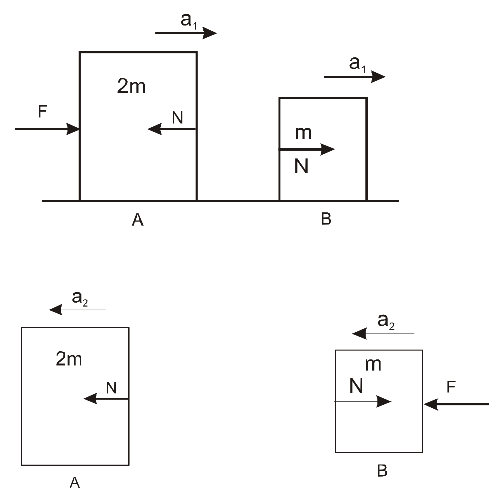
Force exerted by string is always along the string and of pull type.

When there is a contact between a point and a surface the normal reaction is perpendicular to the surface and of push type.

Q.4 (2)

F – N = 2 ma, [Newton`s II law for block A]

$N = ma_1$ [Newton`s II law for block B] ⇒ $N = \frac{F}{3}$

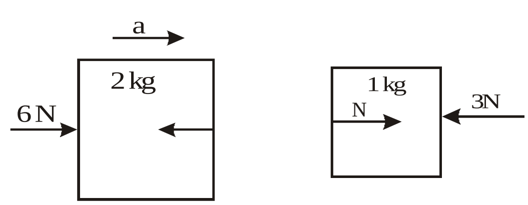


$N = 2\,ma_2$ [Newtons II law for block A]

$F - N = m_2 a$ [Newtons II law for block B]

⇒ N = 2F/3 so the ratio is 1 : 2

Q.5 (3)



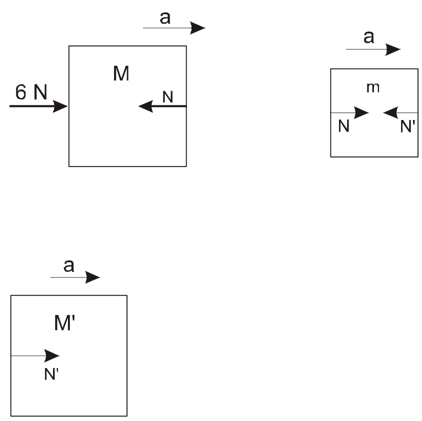
Both blocks are constrained to move with same acceleration.

6 – N = 2a [Newtons II law for 2 kg block]

N – 3 = 1a [Newtons II law for 1 kg block]

⇒ N = 4 Newton

Q.6 (2)



F – N = Ma [Newtons law for block of mass M]

N – N' = ma [Newtons law for block of mass m]

N' = M'a [Newtons law for block of mass M']

$\Rightarrow$ $N' = M' \dfrac{F}{M+m+M'}$

$N = (m + M') \dfrac{F}{M+m+M'} \Rightarrow N > N'$

Q.7 (4)

$\vec{F} = m\vec{a}$

$\vec{a} = \dfrac{d\vec{v}}{dt}$

Q.8 (1)

$\vec{F} = m\vec{a}$

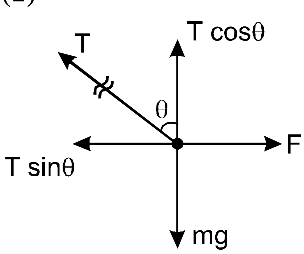
Q.9 (3)

$\vec{F} = m\vec{a}$

Q.10 (2)

In free fall gravitation force acts.

Q.11 (2)

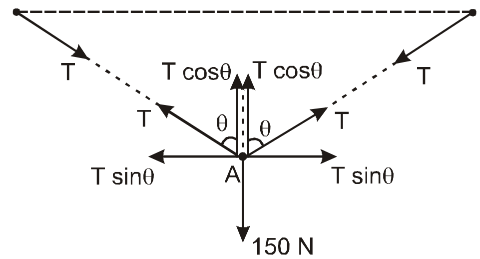


Point A is mass less so net force on it most be zero otherwise it will have ∞ acceleration.

$\Rightarrow$ F – Tsinθ = 0 [Equilibrium of A in horizontal direction]

$\Rightarrow$ $T = \dfrac{F}{\sin\theta}$

Q.12 (4)



T cosθ + T cosθ – 150 = 0 [Equilibrium of point A]

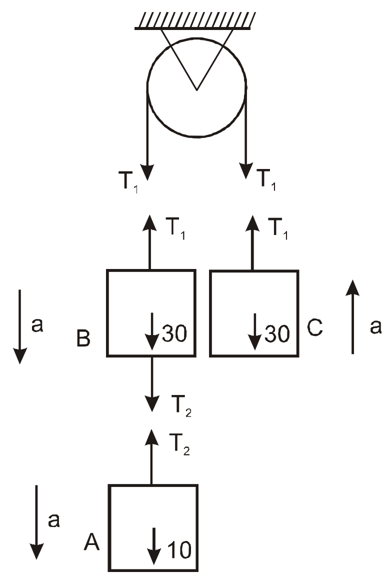
2 T cosθ = 150

$T = \dfrac{75}{\cos\theta}$

When string become straight θ becomes 90°

$\Rightarrow T = \infty$

Q.13 (3)



$10 - T_2 = 1\,a$ [Newton's II law for A]

$T_2 + 30 - T_1 = 3\,a$ [Newton's II law for B]

$T_1 - 30 = 3a$ [Newton's II law for C]

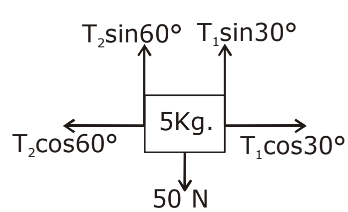
$\Rightarrow$ $a = \dfrac{g}{7}$

$\Rightarrow$ $T_2 = \dfrac{6g}{7}$

Q.14 (2)

From horizontal equilibrium

$\dfrac{T_2}{2} = \dfrac{T_1\sqrt{3}}{2}$



$T_2 = \sqrt{3}T_1$

From vertical equilibrium

$$\frac{T_2\sqrt{3}}{2} + \frac{T_1}{2} = 50$$

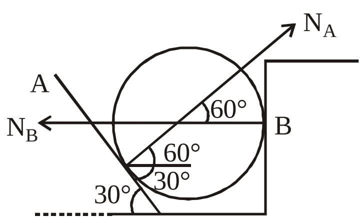
$\Rightarrow \quad T_1 = 25N, \; T_2 = 25\sqrt{3}N$

Q.15 (2)

Component of force in y direction is

$N_A \sin 60° = 500$

$$N_A = \frac{1000}{\sqrt{3}}$$

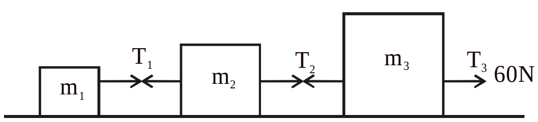


Component of force in x direction is

$N_A \cos 60° = N_B$

$$\Rightarrow \; N_B = \frac{500}{\sqrt{3}}$$

Q.16 (3)



Take a system $(m_1 + m_2 + m_3)$

$T_3 = (m_1 + m_2 + m_3)\, a$

$60 = 60\, a$

$$a = \frac{60}{60} = 1 m/s^2$$

For body m_3

$T_3 - T_2 = m_3 a$

$60 - T_2 = 30$

$T_2 = 30N$

Q.17 (2)

$T = mg$ ...(i)

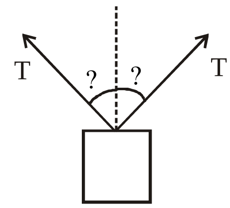
$2T \cos \theta = Mg$ ...(ii)

From equation (i) and (ii)

$\Rightarrow 2mg \cos \theta = Mg$

θ always > 0 so $M < 2m$

Q.18 (3)



(A) $2T = W$, $T = W/2$

(B) $W = 2T \cos \theta$

$$T = \frac{W}{2\cos\theta}$$

In (C) option θ is greater so

$\sec\theta \uparrow$, $T \uparrow$

If tension is more then string may be break-

Q.19 (1)

Relative acceleration Man and car is zero during the journey

$N = 0$

Q.20 (1)

At t = 2 sec $\Rightarrow a = \frac{10}{2} = 5 \text{ m/s}^2$

So, $F = ma = \frac{50}{1000} \times 5 = 0.25\,N$

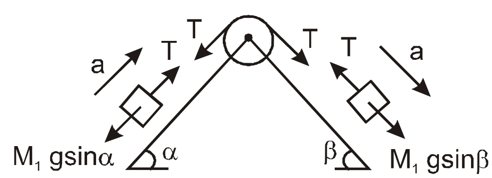
At t = 4 sec

a = 0 So F = 0

$\Rightarrow$ At t = 6 sec,

$\Rightarrow \; a = -5 \text{ m/s}^2 \;\; \Rightarrow F = -0.25$ N

Q.21 (3)



$M_2 g \sin\alpha - T = M_2 a$ [Newton's II law for M_2]

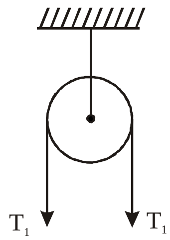
$T - M_1 g \sin\beta = M_1 a$ [Newton's II law for M_1]

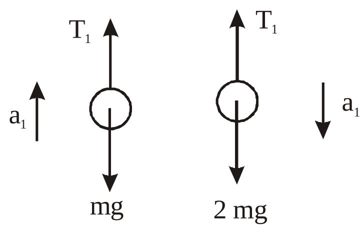
By adding both equations

$$a = \left[\frac{M_2 \sin\alpha - M_1 \sin\beta}{M_1 + M_2}\right] g$$

Q.22 (3)

Case 1



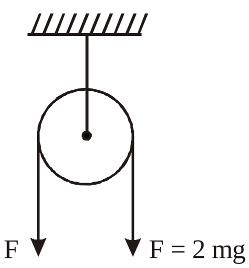


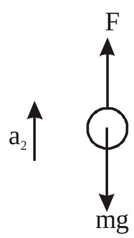
$T_1 - mg = ma_1$ [Newton's II law for m]

$2\,mg - T_1 = 2\,ma_1$ [Newton's II law for 2m]

$\Rightarrow\ a_1 = \dfrac{g}{3}$

Case 2

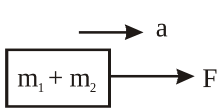




$F - mg = ma_2$[Newton's II law for m]

$\Rightarrow\ 2\,mg - mg = ma_2 \qquad \Rightarrow a_2 = g \Rightarrow a_2 > a_1$

Q.23 (3)



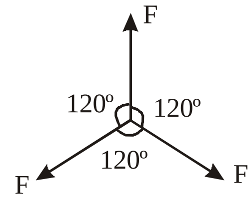
$F = m_1\,4$ [Newton's II law for m_1]

$F = m_2 6$ [Newton's II law for m_2]

$F = (m_1 + m_2)a$ [Newton's II law for $(m_1 + m_2)$]

$\Rightarrow\ F = \left[\dfrac{F}{4}+\dfrac{F}{6}\right]a \Rightarrow 1 = \left[\dfrac{1}{4}+\dfrac{1}{6}\right]a \Rightarrow a = 2.4\ m/s^2.$

Q.24 (4)



Due to symmetry we can say net force on body M is O.

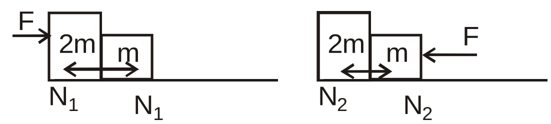
$\therefore$ acceleration is O.

Q.25 (3)

$mg - \dfrac{3}{4}\,mg = ma$ [Newton's II law for man]

$\Rightarrow\ a = \dfrac{g}{4}$

Q.26 (2)

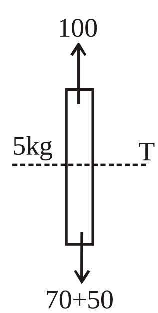


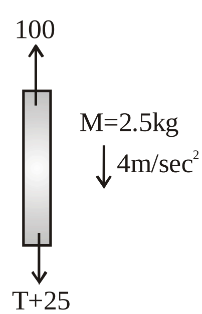
Acceleration $= \dfrac{F}{3m}$

contact force $N_1 = \dfrac{mF}{3m} = \dfrac{F}{3}$

$N_2 = \dfrac{2mF}{3m} = \dfrac{2}{3}F \Rightarrow \therefore N_1 : N_2 = 1 : 2$

Q.27 (2)





$120 - 100 = 5a$ ....(i)

$a = \frac{20}{5} \Rightarrow a = 4 m/s^2$

$T + 25 - 100 = 2.5 \times 4$ ....(ii)

$T = 85$ N

Q.28 (3)

$T - mg = ma$ ...(1)

$Mg - T = Ma$ ...(2)

from (1) and (2)

$a = \left(\frac{M-m}{M+m}\right)g$

Put $M >> m$

$\Rightarrow a = g$

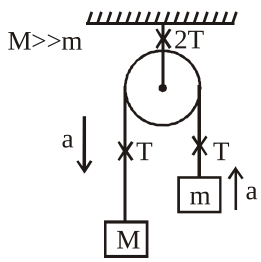
$\therefore T = 2$ mg,

$2T = 4mg$

Q.29 (2)

$\vec{F} = 6\hat{i} - 8\hat{j} + 10\hat{k}$

$\vec{F} = m\vec{a}$



$|\vec{F}| = m|\vec{a}|$

$\sqrt{6^2 + 8^2 + 10^2} = m\ 1$ $\quad m = 10\sqrt{2}$ kg.

Q.30 (4)

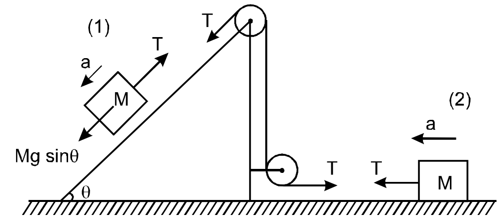
$\upsilon^2 = \upsilon_0^2 + 2as = 1^2 + 2\frac{F}{m}x$

$x = \frac{-m}{2F}\ \upsilon^2 = \upsilon_0^2 + 2as$

$O^2 = 3^2 + \frac{2F^1}{m} \times\ 0 = 9 + \frac{2F^1}{m}\left(\frac{-m}{2F}\right)$

$\Rightarrow F^1 = 9F$

Q.31 (3)



$Mg \sin\theta - T = Ma$ [Newton's II law for block 1]

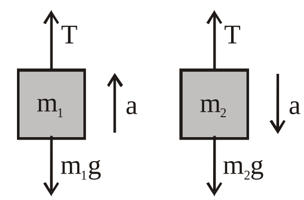
$T = Ma$

[Newton's II law for block 2]

By dividing both equations

$2T = Mg\sin\theta\ T = \frac{Mg\sin\theta}{2}$

Q.32 (3)



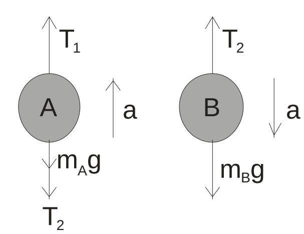
$T - m_1g = m_1a$ ....(i)

$m_2g - T = m_2a$ ....(ii)

On solving equation (i) and (ii)

$a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)g$

Q.33 (1)



$T_1 - T_2 - m_Ag = m_Aa$

$T_2 - m_Bg = m_Ba$

$T_1 - (m_A + m_B)g = (m_A + m_B)$

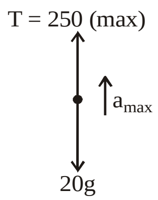
$T_2 = m_B(a + g)$

$\frac{T_1}{T_2} = \frac{m_A + m_B}{m_B}$

Q.34 (2)

Q.35 (4)

$T_{max} = m_{max}g = 25 \times 10 = 250$



$250 - 200 = 20\ a_{max}$

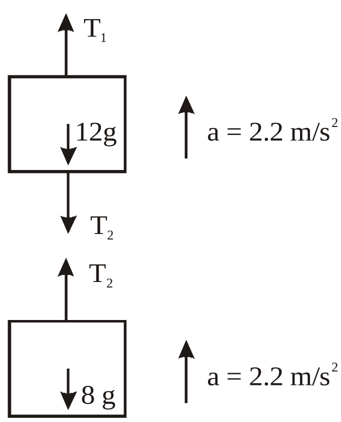
$a_{max} = 2.5\ m/s^2$

Q.36 (1)

$\theta = \downarrow\ \Rightarrow \sin\theta \downarrow$

$mg \sin\theta \downarrow$

Q.37 (3)



$T_2 - 8g = 8a$ [Newton's II law for 8 kg block]

$\Rightarrow\ T_2 = 8 \times 2.2 + 8 \times 9.8 = 96$ N

$T_1 - 12\ g - T_2 = 12\ a$

$\Rightarrow\ T_1 = 12 \times 2.2 + 12 \times 9.8 + 96$

$T_1 = 240$ N

Q.38 (1)

$-v_A - v_A - v_A + v_B = 0$

From constrained

$-5 - 5 - 5 + v_B = 0$

$v_B = 15$ m/s $\downarrow$

Q.39 (1)

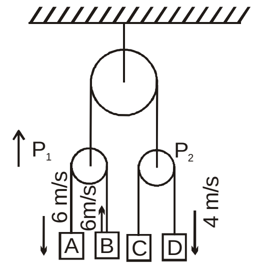
From constrained

$+2 - v_B - v_B + 1 = 0$

$v_B = 3/2\ m/s \uparrow$

Q.40 (2)

$$v_{p_1} = \frac{-6+6}{2} = 0$$



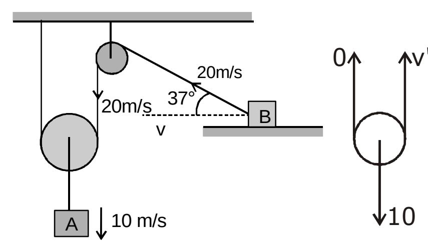
$\Rightarrow\ |v_{p_1}| = |v_{p_2}| = 0$

$v_D = -v_C$

$\therefore$ velocity of C is

$= 4$ m/s

Q.41 (1)



$$\frac{v' + 0}{2} = 10$$

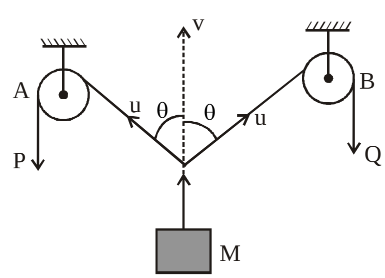
Here Resultant vel. of block 'B' is v

So component of resultant in the direction of v' is

v cos 37° = v' , v cos 37° = 20

$$v = \frac{20 \times 5}{4} = 25\,m/s$$

Q.42 (4)



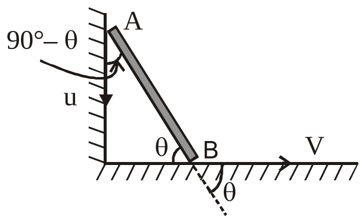
From constrained

The resultant vel. of the Block M is v in vertical direction.

So component of 'v' is direction of u is

$$v \cos\theta = u \Rightarrow v = \frac{u}{\cos\theta}$$

Q.43 (3)

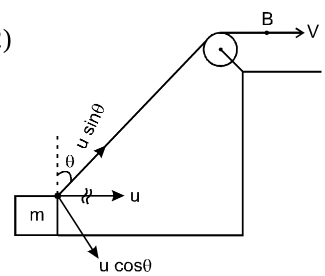


From constrained Motion - (along the rod vel of each particle is same so component of the velocity in the direction of rod is)

$v \cos\theta = u \sin\theta$

$v = u \tan\theta$

Q.44 (2)



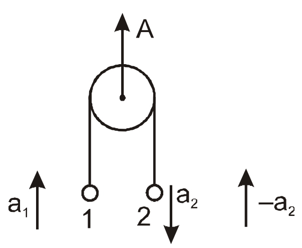
The length of string AB is constant.

$\Rightarrow$ speed A and B along the string are same u sinθ = V

$u \sin\theta = V \quad u = \dfrac{V}{\sin\theta}$

Q.45 (3)

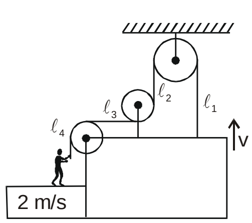
$A = \dfrac{a_1 - a_2}{2}$



Q.46 (2)

$\ell_1 + \ell_2 + \ell_3 + \ell_4 = C$

$$\frac{d\ell_1}{dt} + \frac{d\ell_2}{dt} + \frac{d\ell_3}{dt} + \frac{d\ell_4}{dt} = 0$$



$-v - v + 0 + v + 2 = 0$

$v = 2\text{m/s}$

Q.47 (2)

Let $AB = \ell$, $B = (x , y)$

$\vec{v}_B = v_x\hat{i} + v_y\hat{j}$

$\vec{v}_B = \sqrt{3}\hat{i} + v_y\hat{j} \rightarrow$ (i)

$x^2 + y^2 = \ell^2$

$2x\, v_x + 2y\, v_y = 0$

$\Rightarrow \sqrt{3} + \dfrac{y}{x} v_y = 0$

$\Rightarrow \sqrt{3} + (\tan 60^0)\, v_y = 0$

$\Rightarrow v_y = -1$

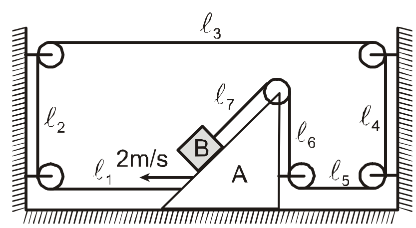
Hence from (i)

$\vec{v}_B = \sqrt{3}\hat{i} - \hat{j}$

Hence

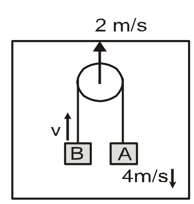
$v_B = 2$ m/s

Q.48 (3)



ℓ_2, ℓ_3, ℓ_4, ℓ_6 are remain constant. $\ell_1 + \ell_5$ = constant so, ℓ_7 will also be constant, so, only velocity of B is along horizontal = 2 m/s

Q.49 (4)

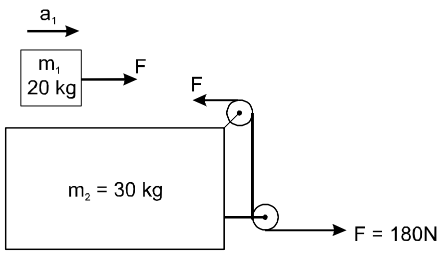


V = (velcoity of B w.r.t ground)

$\dfrac{V - 4}{2} = 2V = 8$ m/s (velcoity of B w.r.t ground)

V' = 6 m/s (velcoity of B w.r.t lift)

Q.50 (1)



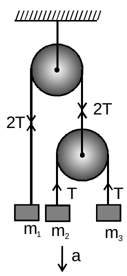
$F = m_1a_1$ [Newton's II law for m_1]

$180 = 20\ a_1$

$\Rightarrow\ a_1 = 9\ m/s^2$

Net force on m_2 is 0 therefore acceleration of m_2 is 0.

Q.51 (3)



$2T = m_1\ g$ ...(1)

$m_2g - T = m_2a$ ...(2)

$T - m_3g = m_3\ a$ ...(3)

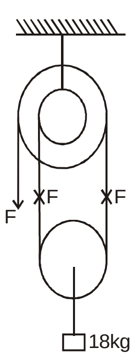
on solving $\dfrac{4}{m_1} = \dfrac{1}{m_2} + \dfrac{1}{m_3}$

Q.52 (2)

18kg at rest => 180 = 2F

F = 90N

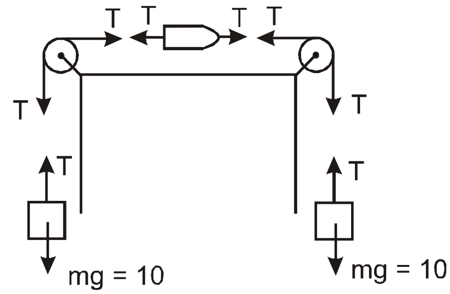
Q.53 (3)



(a) $T = mg + ma$ (2) $T = mg - ma$

$T = mg$

Q.54 (1)



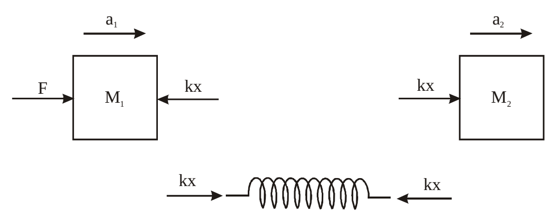
$T - mg = 0$ [Equilibrium of block]

$T - 10 = 0$

$T = 10$

Reading of spring balance is same as tension is spring balance.

Q.55 (4)



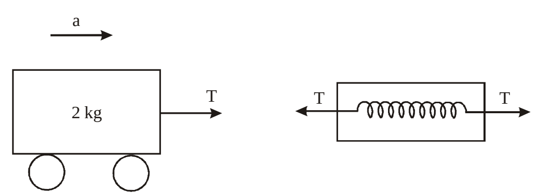
$F - k\,x = m_1\ a_1$ [Newton's II law for M_1]

$kx = m_2a_2$ [Newton's II law for M_2]

By adding both equations.

$F = m_1a_1 + m_2a_2 \Rightarrow a_2 = \dfrac{F - m_1\,a_1}{m_2}$

Q.56 (3)



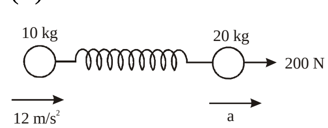
Reading of spring balance is same as tension in the balance.

$\Rightarrow\ T = 10\ g\ = 98\ N$

$T = 2\ a$ [Newton's II law for 2 kg block]

$\Rightarrow\ a = 49\ m/s^2$

Q.57 (2)



$F = m_1a_1 + m_2a_2$ [Newton's law for system]

$200 = 10 \times 12 + 20 \times a$

$a = 4\ m/s^2$.

Q.58 (3)

Q.59 (2)

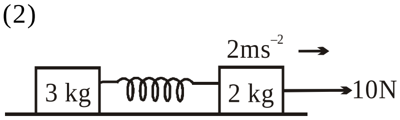
$$T = \frac{2m_1m_2g}{(m_1+m_2)} \Rightarrow T = \frac{2\times5\times1\times10}{6} = \frac{50}{3}$$

$$2T = \frac{100}{3} \approx 33.3kg$$

The spring balance reads

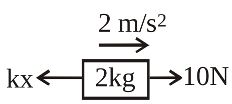
$2T = 33.33kgwt < 60kgwt$

Q.60 (2)



$10 - kx = 2 \times 2$

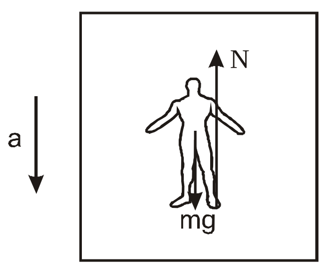
$kx = 6\ N$



$\therefore$ Acceleration of 3 kg = $\frac{6}{3}$ = 2 m/s^2

Q.61 (4)

Weight of man in stationary lift is mg.



$mg - N = ma$ [Newton's II law for man]

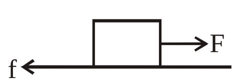
$\Rightarrow N = m(g - a)$

Weight of man in moving lift is equal to N.

$$\Rightarrow \frac{mg}{m(g-a)} = \frac{3}{2} \Rightarrow a = \frac{g}{3}$$

Q.62 (1)

$F < f_{smax}$



friction=F

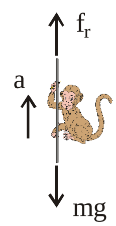
For $F > f_{max}$

friction constant

Q.63 (1)

Q.64 (1)

Monkey is moving up due to friction force



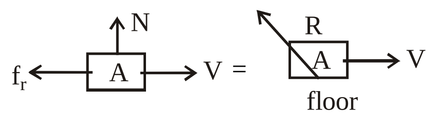
$f_r - mg = ma$

$f_r = m(a+g)$

towards up.

Q.65 (3)

Floor will provide the normal force and friction force the net reaction is provide by the floor is R.



Q.66 (4)

$m_A g\sin 30 = \mu m_A\ g\cos 30$

$m_B\ g\sin 40 = \mu m_B g\cos 40$

Does not depend on mass so all three are possible.

Q.67 (2)

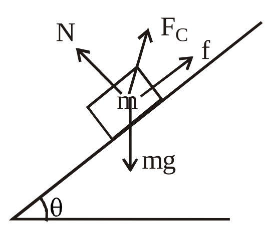
$f_{max} > mg \sin\theta$

$\theta\uparrow\ \sin\theta\uparrow$

at this condition block remains rest when

$mg \sin\theta > f_{max}$

sliping slant



For θ < angle of repose

$F_c = mg$

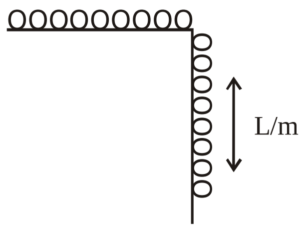
For θ > angle of repose

as $\theta \uparrow$ $f = \mu mg\cos\theta \downarrow$

$N = mg\cos\theta \downarrow$

Q.68 (2)

$$\mu\lambda L\left(1-\frac{1}{n}\right)g = \lambda\frac{L}{n}g$$



$$\mu = \frac{1}{n-1}$$

Q.69 (2)

$f_{max} = \mu\, mg\cos\theta$

$$f_{s_{max}} = 0.7\times2\times9.8\times\frac{\sqrt{3}}{2} = 7\sqrt{3}$$

$mg\sin\theta = 9.8$

As $mg\sin\theta < f_{smax}$ so friction requird is $mg\sin\theta$.

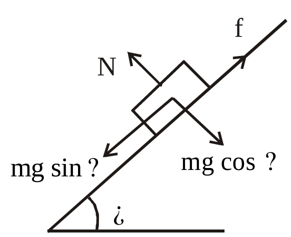
Q.70 (1)

$N = mg\cos\theta$

$f_s \le \mu N$

$mg\sin\theta \le \mu\, m\, g\cos\theta$

$\mu \ge 1$



Q.71 (4)

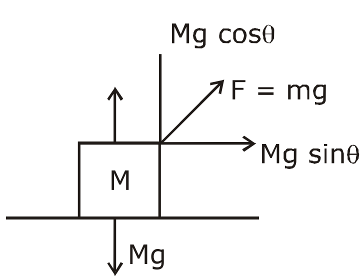
$N = Mg - Mg\cos\theta$, $f_{max} = \mu N$

$f_{max} = \mu Mg(1-\cos\theta)$

$Mg\sin\theta \ge f_{max}$

$Mg\sin\theta \ge \mu Mg(1-\cos\theta)$

$\mu \le \sin\theta/(1-\cos\theta)$



$$\mu \le \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}$$

$$\mu \le \tan\frac{\theta}{2}$$

Q.72 (1)

Friction not depend on surface Area so angle remain same.

$\therefore$ Angle = 30°

Q.73 (3)

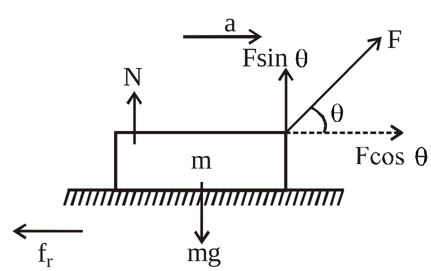
$v = u + at \Rightarrow a_A = -\mu g$

$a_B = -\mu g \Rightarrow a$ = same $\Rightarrow$ u = same

Time taken to stop is also same

Does not depend on mass

Q.74 (3)



$F\sin\theta + N = mg$

or $N = mg - F\sin\theta$ ...(1)

$f_r = \mu N$ ...(2)

$F\cos\theta - f_r = ma$ ...(3)

on solving (1), (2) & (3)

$$a = \frac{F\cos\theta - \mu(mg - F\sin\theta)}{m}$$

$$a = \frac{F}{m}(\cos\theta + \mu\sin\theta) - \mu g$$

Q.75 (1)

move with a constant velocity

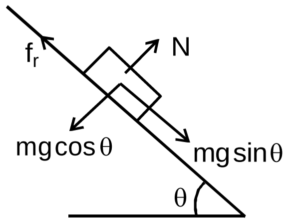
So $ma = m\mu g$ (in negative direction)

$a = \mu g$

$\Rightarrow$ $v^2 - u^2 = 2as$ $v_f^2 = v_i^2 + 2as$

$v = \sqrt{2\mu gs}$ here $v_f = 0$, $v_i = v$

Q.76 (1)



Let length is ℓ of inclined plane, then

$f_r = \mu N = \mu\, mg \cos\theta$ (when friction is present)

$mg \sin\theta - f_r = ma$

$a = g(\sin\theta - \mu \cos\theta)$

$mg \sin\theta - \mu\, mg \cos\theta = ma$ ...(1)

Now $\ell = \frac{1}{2} a t_1^2 = \frac{1}{2} g(\sin\theta - \mu\cos\theta) t_1^2$

Now $\ell_1 = \ell_2 = \ell$

Without friction

$ma = mg \sin\theta$, $a = g \sin\theta$

$\ell_2 = \ell = \frac{1}{2} g \sin\theta\, t_2^2$

$t_1 = 2t_2$, $t_1^2 = t$ and $t_2 = t/2$

$$\frac{1}{2} g(\sin\theta - \mu\cos\theta) t^2 = \frac{1}{2} g(\sin\theta - 0 \times \cos\theta)\left(\frac{t^2}{4}\right)$$

$4(\sin\theta - \mu\cos\theta) = \sin\theta$

$\Rightarrow \mu = \frac{3}{4} = 0.75$

Q.77 (1)

$V^2 = 2 \times g \sin\theta \times l$

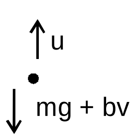
$\frac{v^2}{n^2} = 2 \times (g\sin\theta - \mu g\cos\theta)$

$\sin\theta\left(1 - \frac{1}{n^2}\right) = \mu\cos\theta$

$\mu = \tan\theta\left(1 - \frac{1}{n^2}\right)$

JEE-ADVANCED OBJECTIVE QUESTIONS

Q.1 (B)



In upward motion

as v ↓

Force ↓

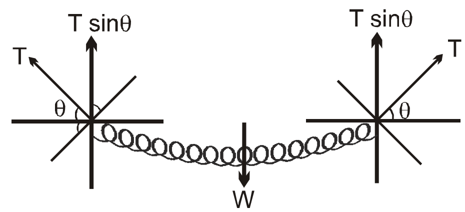
acceleration ↓

and takes less time to reach at top.

Q.2 (B)

F_1 may be equal to F_2

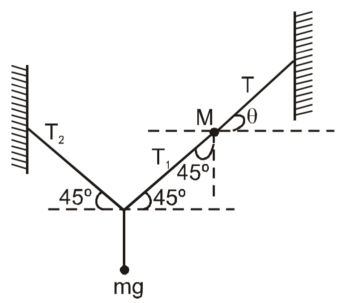
Q.3 (A)



$2T \sin\theta = W$

$T = \frac{W}{2} \operatorname{cosec}\theta$

Q.4 (A)



$T_1 \cos 45^\circ = T_2 \cos 45^\circ$

$\Rightarrow T_1 = T_2$

$(T_1 + T_2) \sin 45^\circ = mg$

$\sqrt{2}\, T_1 = mg$

$T_1 = \frac{mg}{\sqrt{2}}$.

$T \sin\theta = mg + \frac{T_1}{\sqrt{2}}$

$T \sin\theta = mg + \frac{mg}{2}$ ....(i)

$T \cos\theta = \frac{T_1}{\sqrt{2}} = \frac{mg}{2}$ ....(ii)

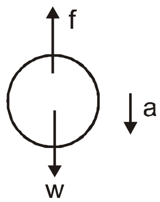
dividing (i) and (ii)

$\tan\theta = \frac{M + m/2}{m/2} = 1 + \frac{2M}{m}$ Ans.

Q.5 (C)

$v = u + a_1 t_1 + a_2 t_2 + a_3 t_3$

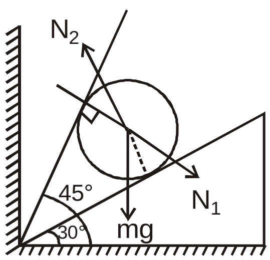
Q.6 (C)

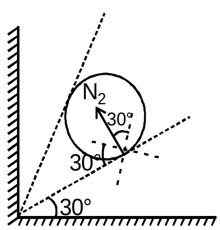


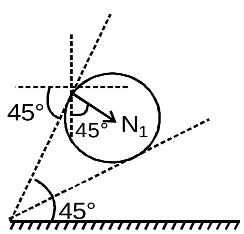
$w - f = ma \; w - ma = g$

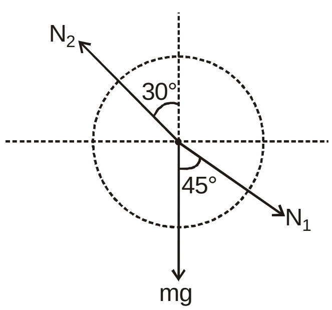
$$w\left\{1-\frac{m}{w}a\right\} = fw\left\{1-\frac{m}{mg}a\right\} = fw\left\{1-\frac{a}{g}\right\} = f$$

Q.7 (A)









In vertical direction

$$50+\frac{N_1}{\sqrt{2}}=\frac{N_2\sqrt{3}}{2} \qquad ...(1)$$

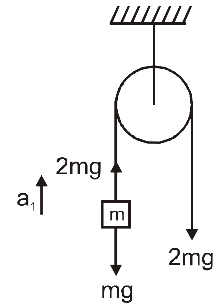
In horizontal direction

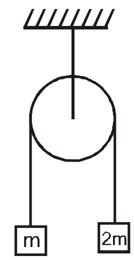
$$\frac{N_1}{\sqrt{2}}=\frac{N_2}{2} \qquad ...(2)$$

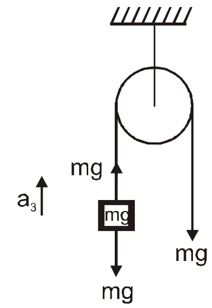
On solving eq^n (1) and (2) we get N_1 and N_2

$N_1 = 96.59$ N, $N_2 = 136.6$ N

Q.8 (B)





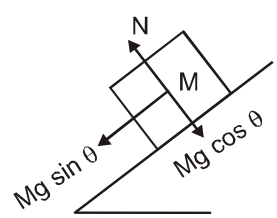


$a_1 = \dfrac{2mg - mg}{m}$ $a_2 = \dfrac{(2m-m)g}{2m+m}$ $a_3 = 0$

$a_1 = g$ $a_2 = \dfrac{g}{3}$

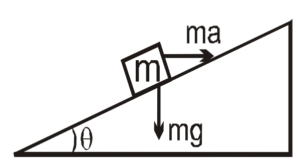
So, $a_1 > a_2 > a_3$

Q.9 (C)



$N = Mg\cos\theta \rightarrow$ force eserted by plane on the block.

Q.10 (B)



$ma\cos\theta = mg\sin\theta$

$a = g\tan\theta$

$N = mg\cos\theta + ma\sin\theta$

$$= mg\cos\theta + \frac{mg\sin^2\theta}{\cos\theta} = \frac{mg}{\cos\theta}$$

Q.11 (A)

legnth of groove = $\sqrt{3^2+4^2}$

= 5m

acceleration along the incline = gsinθ = g sin 30º = g/2

acceleration along the groove = g/2 cos (90–α) = g/2 sinα = $\frac{g}{2}\times\frac{4}{5}$ = 4m/s^2

$v^2 = 2as$

v = $\sqrt{2\times4\times5}$ = $\sqrt{40}$ m / sec .

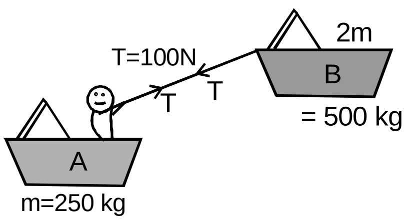
Q.12 (C)

(A) 40 cos 30° = $20\sqrt{3}$ N

(B) weight = 5 kg

(C) Net = zero

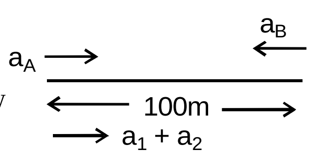
Q.13 (B)



M = 250 kg

$a_A = \frac{F}{m} = \frac{100}{250} = \frac{2}{5}$ and $a_B = \frac{100}{500} = \frac{1}{5}$

Now



$a_{AB} = a_1 + a_2 = \frac{2}{5}+\frac{1}{5}=\frac{3}{5}$

$100 = \frac{1}{2}(a_1+a_2)t^2 \Rightarrow \quad 100=\frac{1}{2}\left(\frac{3}{5}\right)t^2$

t^2 = 333.33

t = 18.25 = 18.3 sec

Q.14 (C)

Given

$\vec{a}+\vec{b}+\vec{c}+\vec{d}+\vec{e}=3\hat{i}$ ...(1)

$\vec{b}+\vec{c}+\vec{d}+\vec{e}=-\hat{i}$ ...(2)

$\vec{a}+\vec{c}+\vec{d}+\vec{e}=24\hat{j}$ ...(3)

(1) – (2) $\Rightarrow \vec{a}=4\hat{i}$

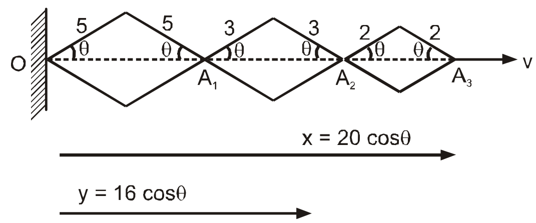
(1) – (3) $\Rightarrow \vec{b}=3\hat{i}-24\hat{j}$

Now $\vec{a}+\vec{b}=7\hat{i}-24\hat{j}$

$|\vec{a}+\vec{b}|=\sqrt{49+(24)^2}=25$

Q.15 (D)

Case–1 seen from ground is same as Case-2 seen from train.



v = $\frac{dx}{dt}$ = –20 sinθ $\frac{d\theta}{dt}$

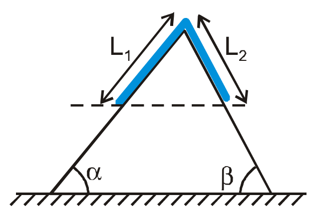
u = $\frac{dy}{dt}$ = –16 sinθ $\frac{d\theta}{dt}$

$\Rightarrow$ u = $\frac{4}{5}$ v = 0.8 v

Q.16 (A)

Let L_1 and L_2 be the portions (of length) of rope on left and right surface of wedge as shown

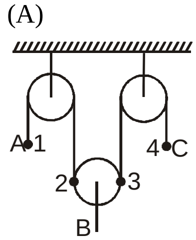
∴ magnitude of acceleration of rope



$a = \dfrac{\frac{M}{L}[L_1\sin\alpha - L_2\sin\beta]g}{M} = 0$

(∵ L_1 sinα = L_2 sinβ)

Q.17 (A)



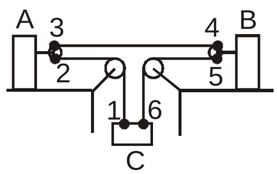
From constrained

$a_1 + a_2 + a_3 + a_4 = 0$

$-a - a_B - a_B + f = 0$

$$a_B = \left(\frac{f}{2} - \frac{a}{2}\right) = \frac{1}{2}(f - a)\uparrow$$

Q.18 (A)



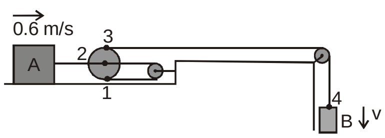
From constrained

$a_1+a_2+a_3+a_4+a_5+a_6 = 0$

$-a_C+2+2-1-1-a_C = 0$

$a_C = 1\ m/s^2 \uparrow$

Q.19 (A)



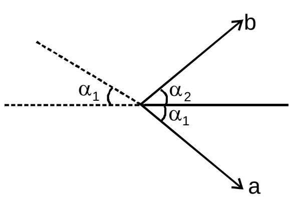
From constrained

$v_1 + v_2 + v_3 + v_4 = 0$

$v - 0.6 - 0.6 - 0.6 = 0$

$V = 1.8$ m/s

Q.20 (A)



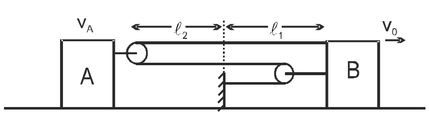
In horizontal direction net acceleration is zero.

So, $b \cos \alpha_2 = a \cos \alpha_1$

$$b = \frac{a\cos\alpha_1}{\cos\alpha_2}$$

Q.21 (B)

By setting string length constant



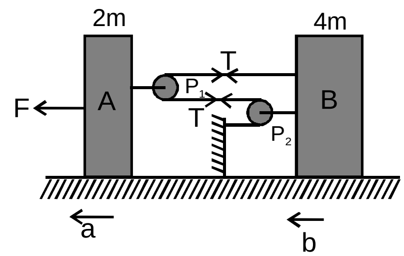
$L = 3\ell_1 + 2\ell_2$

$\Rightarrow 3v_0 = 2v_A$

$v_A = \frac{3}{2}v_0$

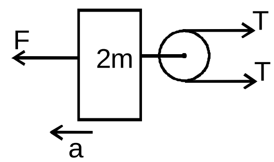
$v_{AB} = v_A - v_B = \frac{v_0}{2}$ towards right.

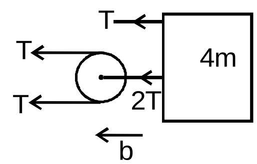
Q.22 (A)



From Constrain equation

$2a = 3b$ ..(1)





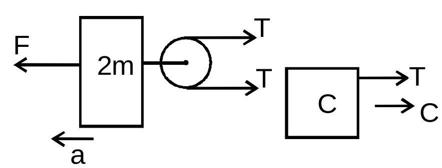
$F - 2T = 2ma$ ...(2)

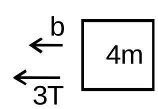
$3T = 4mb$ ...(3)

On solving (1), (2) & (3)

$$b = \frac{3F}{17}$$

Q.23 (B)





constrain equation $2a = 3b + c$ ..(1)

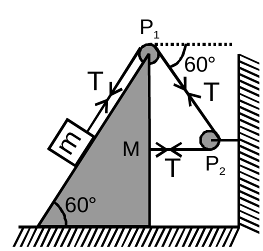
$F - 2T = 2ma$ ..(2)

$T = mc$ ..(3)

$3T = 4mb$ ..(4)

on solving above four equation $b = \frac{3F}{21m}\ m/s^2$

Q.24 (A)

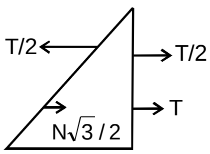


From Constrain equation

$$-b + 0 + 0 - b/2 + b\frac{\sqrt{3}}{2} + a - b\frac{\sqrt{3}}{2} = 0$$

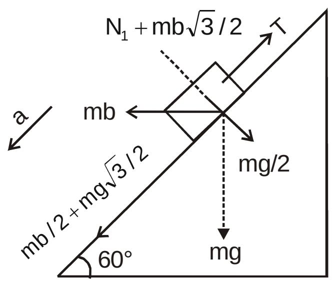
$$a = \frac{3b}{2} \qquad ..(1)$$

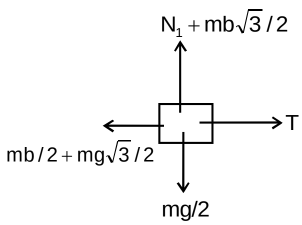
F.B.D of 8 kg block



$$T + \frac{N\sqrt{3}}{2} = 8b \qquad ....(2)$$

F.B.D of 2 kg block





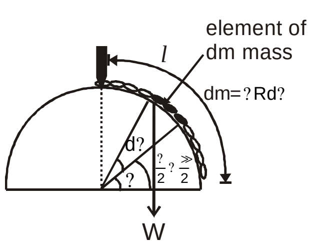
$$mg\frac{\sqrt{3}}{2} + \frac{mb}{2} - T = ma \qquad ...(3)$$

$$N + mb\frac{\sqrt{3}}{2} = \frac{mg}{2} \qquad ...(4)$$

On solving above four equation, we get

$$b = \frac{30\sqrt{3}}{23} \text{ m/s}^2$$

Q.25 (B)



$$F = \int_{\pi/2 - l/r}^{\pi/2} \ell rg \cos\theta \, d\theta$$

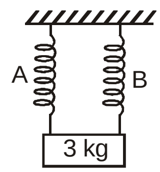
$$F = \lambda rg\left[1 - \cos\frac{\ell}{r}\right]$$

$$a = \frac{m}{l}.\frac{rg}{m}\left[1 - \cos\frac{\ell}{r}\right]$$

$$a = \frac{rg}{\ell}\left[1 - \cos\frac{\ell}{r}\right]$$

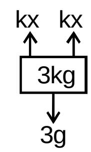
Q.26 (A)

Before cutting

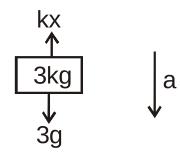


2kx = 3g
kx = 15

After cutting

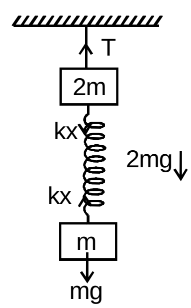


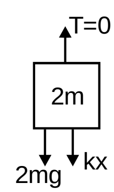
after cut the spring A.



$$a = \frac{3g - kx}{3} = \frac{15}{3} = 5m/s^2$$

Q.27 (B)

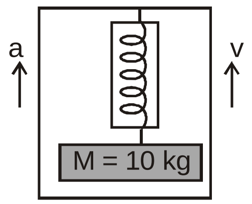




T = Kx + 2 mg ....(i)
Kx = mg ....(ii)
T = 3 mg
After cutting T = 0
downwards net force

$$\therefore \quad a = \frac{3mg}{2m} = \frac{3g}{2}$$

Q.28 (i) A, (ii) A, (iii) A, (iv) C, (v) B, (vi) C, (vii) C, (viii) B

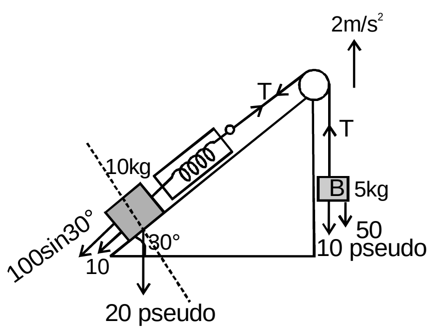


(a) v = 0 or v = constant , a = 0
w = m(g + a)
= 10 (g + 0)
= 100 N

(b) v = 0 or v = constant
a = upward = 2m/s^2
w = m (g + a)
= 120 N

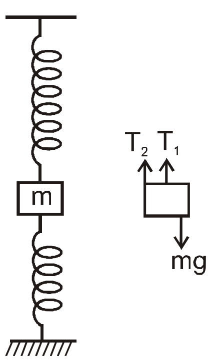
(c) v = 0 or v = constant
a = downward = 2m/s^2
w = m (g – a)
= 80 N

Q.29 (**C**)



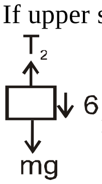
Tension in the string is 60N.
So spring balance reading
= 6 kg or 60 N

Q.30 (**B**)



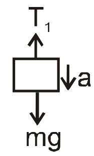
$T_1 + T_2 = mg$

If upper spring is cut



$mg – T_2 = m \times 6$ .....(i)

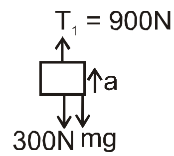
If lower spring is cut :



$mg – T_1 = ma$ ......(ii)

adding (i) and (ii)
$2mg – \{T_1 + T_2\} = m (a + 6)$
$2mg – mg = m (a + 6)$
$mg = m (a + 6)$
$g = a + 6$ $a = 4m/s^2$.

Q.31 (**B**)



$900 – 300 – m \times 10 = ma$ $600 = m (10 + a)$

$$\frac{600}{10+a} = m$$

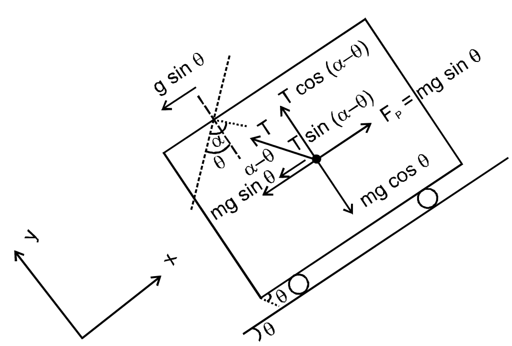
$$\frac{600}{10+10} = m = \frac{600}{20} = 30 \text{ kg.}$$

Q.32 (**C**)
For first case tension in spring will be
$T_s = 2mg$ just after 'A' is released.

$2mg – mg = ma \Rightarrow a = g$
In second case $T_s = mg$

$2mg – mg = 2mb$
$b = g/2$
$a/b = 2$

Q.33 (**A**)



F.B.D. of mass m is w.r.t. trolley

$T \sin(\alpha - \theta) + mg \sin\theta - F_p = 0$ [Equilibrium of mass in x direction w.r.t. trolley]

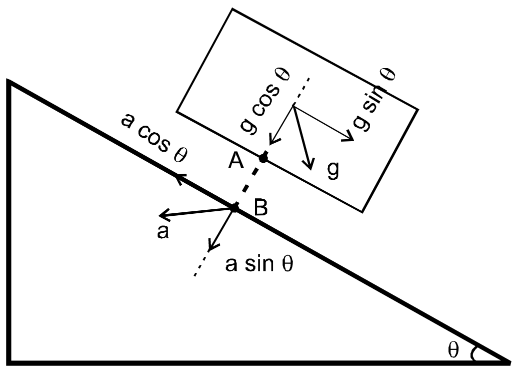
$\Rightarrow$ $T \sin(\alpha - \theta) + mg \sin\theta - mg \sin\theta = 0$

$\Rightarrow$ $T \sin(\alpha - \theta) = 0$

since T cant be zero , $\sin(\alpha - \theta)$ must be zero

$\Rightarrow$ $\alpha = \theta$

Q.34 (**C**)

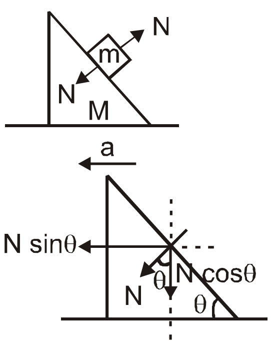


acceleration of point A and B must be some along the line ⊥ to the surface

$\Rightarrow$ $a \sin\theta = g \cos\theta$

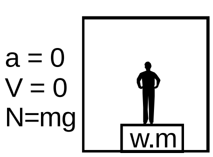
$a = g \cot\theta$

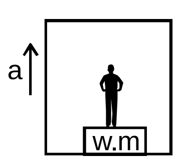
Q.35 (**C**)



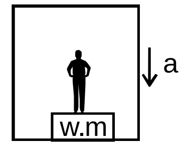
$a = \dfrac{N\sin\theta}{M}$ along (– ve × axis)

Q.36 (i) A, (ii) A, (iii) C, (iv) D, (v) B, (vi) D, (vii) B, (viii) B

(a) 

(b) 

$N = mg + ma$

(c) 

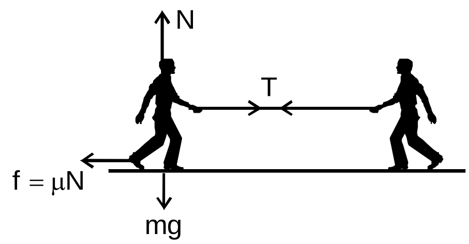
$N = mg - ma$

Independent of the direction of velocity.

Q.37 (B)

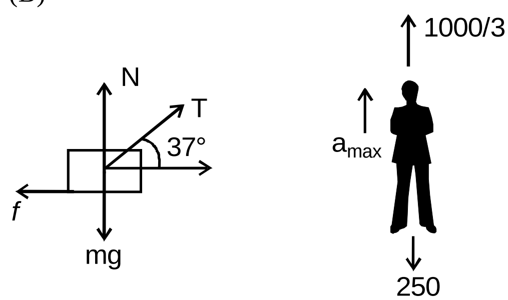
If v = 0 or v = constant then frame is inertial.

Q.38 (B)



Friction force will more then man will not slip.
N is More

Q.39 (B)



$T \cos 37° = f$

$N + T \sin 37° = mg$

$\therefore$ $N = 100\,g - T \sin 37° = 100\,g - \dfrac{3T}{5}$

and $T \cos 37° = \mu N$

$T \cos 37° = \mu(100\,g - \dfrac{3T}{5})$

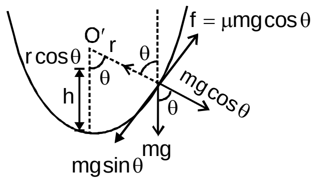
on solving $T = \dfrac{1000}{3}$ $\quad (\mu = \dfrac{1}{3})$

$T - Mg = ma_{max}$

$\dfrac{1000}{3} - 250 = 25 \times a_{max}$

$a_{max} = \dfrac{g}{3} = \dfrac{10}{3}\,m/s^2$

Q.40 (B)



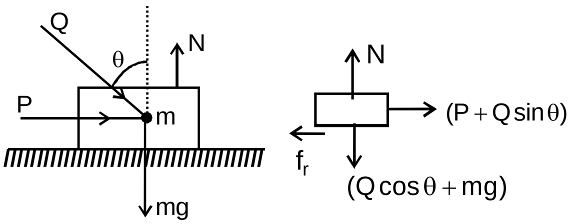
$h = r - r\cos\theta$

$\mu\, mg\cos\theta = mg\sin\theta$

$\tan\theta = u\cos\theta = \dfrac{1}{\sqrt{1+\mu^2}}$

$h = r(1-\cos\theta) = r\left[1 - \dfrac{1}{\sqrt{1+\mu^2}}\right]$

Q.41 (A)

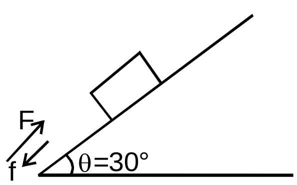


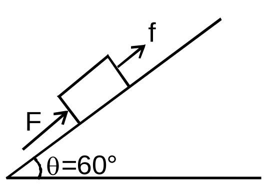
$f_r = \mu N = \mu(mg + Q\cos\theta)$

$f_r = P + Q\sin\theta$

$\mu = \dfrac{(P+Q\sin\theta)}{(mg+Q\cos\theta)}$

Q.42 (C)





$F = mg\sin 30° + \mu\, mg\cos 30°$

$= \dfrac{mg}{2}[1+\mu\sqrt{3}]$

...(1)

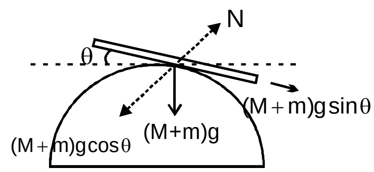
$F + f = mg\sin 60°$

$F = \dfrac{mg}{2}[\sqrt{3}-\mu]$ ...(2)

Now (1) = (2)

$1+\mu\sqrt{3} = \sqrt{3}-\mu \Rightarrow \mu = \dfrac{(\sqrt{3}-1)}{(\sqrt{3}+1)}$

Q.43 (**A**)



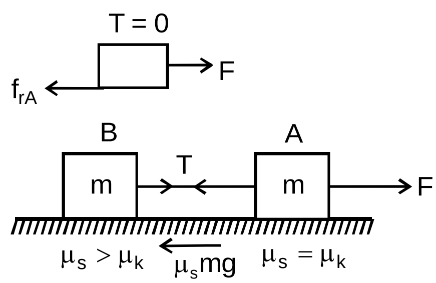
For equilibrium condition

$(M+m)g\sin\theta = \mu(M+m)g\cos\theta$

$\tan\theta = \mu$

Here $\mu \rightarrow$ coefficient of friction between board & log.

Q.44 (**A**)



Initially

$F - f_{r_A} = 0 \Rightarrow t - \mu_s\, mg = 0 \Rightarrow t = \mu_s\, mg$

[till or $f_{rB} = \mu_s mg$, $t - \mu_s mg = \mu_s\, mg$

$t = 2\,\mu_s\, mg$]

$T = F - f_{rA} = f_{rB}$

$T = t - \mu_s mg = f_{rB}$

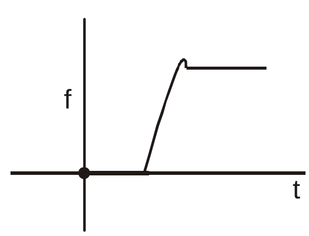
$t = \mu_s mg$ block be will not move

$\mu_s mg < t \le 2\mu_s mg$ block be will not move, static friction will work

after $t > 2\mu_s mg$ kinetic friction will work

$a = \dfrac{F - \mu_s\, mg - \mu_k\, mg}{m}$

So $T = F - \mu_s\, mg - ma$ after $t = 2\mu_s\, mg$

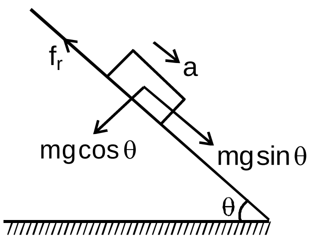


Q.45 **(B)**

$$\frac{1}{2}mv_0^2 = \mu mgL$$

$$v_0 = \sqrt{2\mu gL}$$

Q.46 **(A)**



mg sin θ – f_r = ma

⇒ mg sin θ – μ mg cos θ = ma

g[sinθ – μ cos θ] = a

$$a = \frac{vdv}{dx}$$

$$\Rightarrow \frac{vdv}{dx} = g[\sin\theta - \mu_0 x\cos\theta] \quad [\therefore\ \mu = \mu_0 x]$$

$$\int_0^x g[\sin\theta - \mu_0 x\cos\theta]dx = \int_0^v vdv$$

[Here v = 0]

[Here initial & final velocity is zero]

$$\therefore\ g[\sin\theta x - \mu_0\frac{x^2}{2}\cos\theta] = 0$$

$$\Rightarrow x = \frac{2}{\mu_0}\tan\theta$$

Q.47 **(A)**

$$\int_0^{x/2} g(\sin\theta - \mu_0 x\cos\theta)\,dx = \int_0^v vdv$$

$$g[\sin\theta.\frac{x}{2} - \frac{\mu_0}{2}\left(\frac{x}{2}\right)^2\cos\theta] = \frac{v^2}{2}$$

Keeping the value

$$x = \frac{2}{\mu_0}\tan\theta \Rightarrow v = \sqrt{\frac{g\tan\theta\sin\theta}{\mu_0}}$$

Q.48 **(D)**

a = g sinθ - μ g cosθ

At the x increases, u ↑ a ↓

so when a = 0 instant give maximum speed

g sin 37° – (0.3) xg cos 37° = 0

$$6 - \frac{3}{10}\times x\times 8 = 0$$

$$\Rightarrow \quad x = \frac{60}{3\times 8} = \frac{20}{8} = 2.5\,m$$

Q.49 **(a) B, (b) D, (c) A**

(a) T – mg sin 45° = ma

$$T - \frac{mg}{\sqrt{2}} = \text{Given } a = \frac{g}{5\sqrt{2}}$$

$$T = \frac{6}{5}\frac{mg}{\sqrt{2}}$$

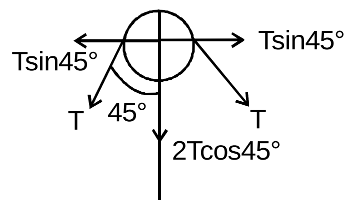
(b) 3 mg sin 45° – T – μN = 3ma

$$\frac{3mg}{\sqrt{2}} - \frac{6mg}{5\sqrt{2}} - \frac{3mg}{5\sqrt{2}} = \mu\ (3\text{ mg cos }45°)$$

$$3mg\left[\frac{1}{\sqrt{2}} - \frac{2}{5\sqrt{2}} - \frac{1}{5\sqrt{2}}\right] = 3mg\times\frac{\mu}{\sqrt{2}}$$

$$\mu = \frac{2}{5}$$

(c)

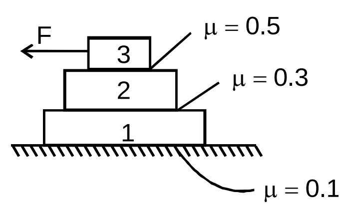


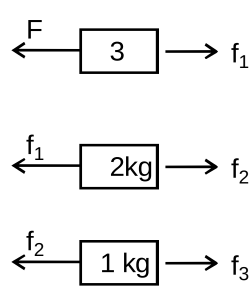
So 2T cos 45° = F

$$2\times\frac{6mg}{5\sqrt{2}}\times\frac{1}{\sqrt{2}} = F$$

$$\therefore\ F = \frac{6mg}{5}\ \text{downward}$$

Q.50 **(C)**



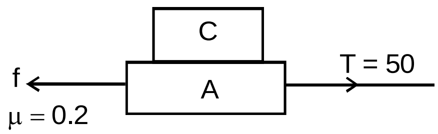


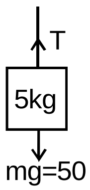
$f_{1\,max}$ = 15 N, $f_{2\,max}$ = 15 N, $f_{3\,max}$ = 6 N

Q.51 **(A)**

System is at rest contract

So,





At rest

$f = T = \mu N$

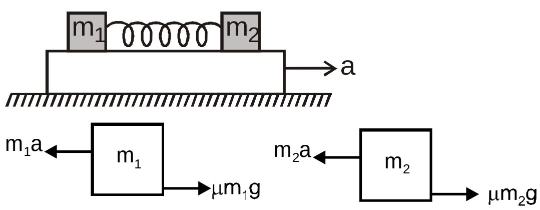
$N = 50/0.2 = 250$ newton

$m_A g + m_c g = 250$

$10\times10 + m_c \times 10 = 250$

so $m_c = 15$ kg

Q.52 **(D)**



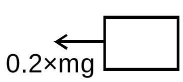
$a_{net} = a - \mu g$ $a_{net} = a - \mu g$

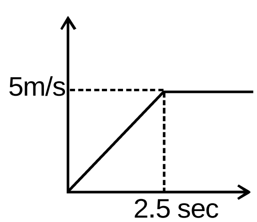
$\therefore$ f_r static and f_r kinetic both provide same acceleration to m_1 and m_2.

So no relative motion between them

$\therefore$ x = 0 (Always)

Q.53 (C)





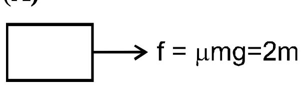
For t < 1 sec $\therefore$ $a_B = 2$ m/s^2

and velocity of truck is 5 m/s

$\therefore$ Friction will act after 1 sec due to relative motion between block and truck

$5 = 2 \times t$, t = 2.5 sec.

Q.54 **(A)**



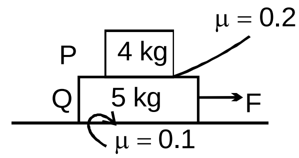
$\{a = \mu g = 0.2 \times 10 = 2\}$

acceleration = 2 m/s^2

So, $4 = 2 \times t$ $\Rightarrow$ t = 2 sec

$\therefore$ $S = \frac{1}{2}.2.(2)^2 = 4$ m

Q.55 **(C)**



$f_1 = 0.2 \times 40 = 8$ N

$f_2 = 0.1 \times 90 = 9$ N

Max. acceleration for system $a = \frac{8}{4} = 2$ m/s^2

Minimum force needed to cause system to move = 9 N

JEE-ADVANCED

MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 (A, B, C, D)

Newton's III[rd] Law.

Q.2 (A, B, C)

$F = \alpha t$

$a = \frac{dv}{dt} = \frac{\alpha}{m}\ t$

....(i)

straight line curve 1 $dv = \frac{\alpha}{m}\ tdt$

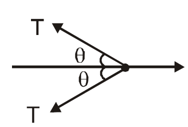
$v = \frac{\alpha}{m}\ \frac{t^2}{2}$ curve 2

...(ii)

divide (ii) by

$v = \frac{t}{2}\ a = \frac{a}{2}\times\frac{am}{\alpha} = \frac{a^2 m}{2\alpha} \rightarrow$ Paacebole curve 2.

Q.3 (A,C)

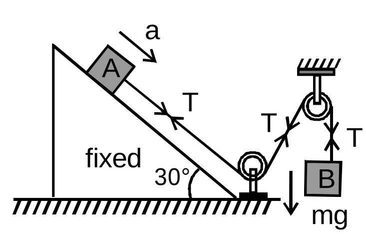


$F = 2\,T\cos\theta$ $T = \frac{F}{2\cos\theta}$

$\theta\uparrow$ cos $\theta\downarrow$ $T\uparrow$

on incrcasing θ, cosθ decreases and hence T increases.

Q.4 (B,D)

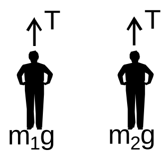


$T + mg \sin \theta = ma$ ....(1)
$mg - T = ma$ ....(2)
on solving (1) & (2)

$a = \frac{3g}{4} \qquad T = \frac{3g}{4}$

Q.5 (A,B,C)
Slope of x – t curve gives velocity
In region AB, BC, CD have constant slope.
$\Rightarrow a = 0$
$\Rightarrow$ net force = 0

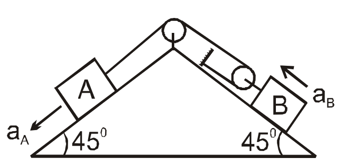
Q.6 (A,B,D)



(A) $T = m_1g < m_2g$
$\therefore$ Acceleration of m_2 is $\downarrow$
(B) $T = m_2 g > m_1g$
$\therefore$ acceleation of m_1 is $\uparrow$
(C) Masses is different
$\therefore$ Not possible
(D) $T - m_1g = m_1a$
$m_2g - T = m_2a$

on solving $a = \frac{(m_2 - m_1)g}{(m_1 + m_2)}$ Possible

Q.7 (A, B, D)



By string constraint

$a_A = 2a_B$ ....(1)

equation for block A.

$10 \times 10 \times \frac{1}{\sqrt{2}} - T = 10\, a_A$ ......(2)

equation for block B.

$2T - \frac{400}{\sqrt{2}} = 40\, a_B$ ....(3)

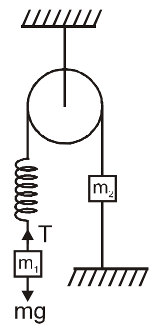
Solving equation (1) , (2) & (3) we get

$a_A = \frac{-5}{\sqrt{2}}$ m/s^2

$a_B = \frac{-5}{2\sqrt{2}}$ m/s^2

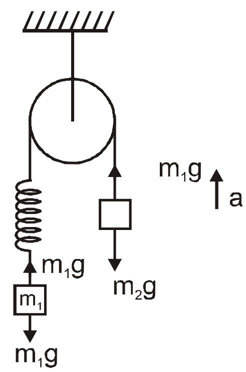
$T = \frac{150}{\sqrt{2}}$ N

Q.8 **(A, C)**

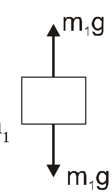


$T = m_1g$
when thred is burnt, tension in spring remains same $= m_1g$.

$m_1g - m_2g = m_2a \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g = a$ = upwards



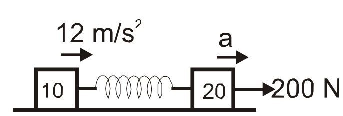
for m_1



a = o

Q.9 **(A, C)**
$T = mg_{eff} = w_{eff}$
$= 5\,(10 + 2)$
$= 60$ N
$= 6$ kg f

Q.10 (B,C)



Apply NLM on the system

$200 = 20\,a + 12 \times 10$

$\frac{80}{20} = a = 4\ m/s^2$

spring Force $= 10 \times 12 = 120$ N

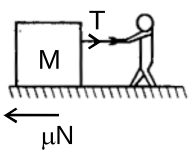
Q.11 (C, D)
Pseudo force depends on acceleraton of frame and mass of object

Q.12 (B, D)
$f_c = N$ (Given)

$\therefore f_c = \sqrt{N^2 + f^2}$

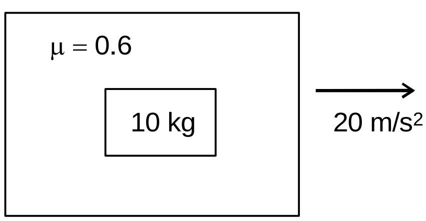
Acceleration to condition $f = 0 \Rightarrow f_c = N$

Q.13 (**A,B,C**)



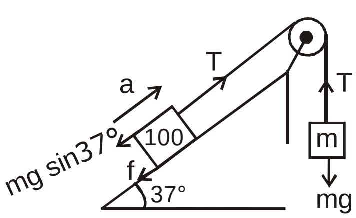
$T - \mu N = ma$
As T ↑ man
Can have tendency
to move

Q.14 (**A,B,C,D**)



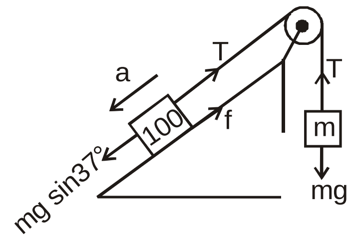
(A) Acceleration of box = 20 m/s^2
(when consider as system)
Force on Box
F = 200 N
N = 200 N
$f_{max} = \mu N = 0.6 \times 200 = 120$ N
(B) $f_{required} = mg = 10 \times 10 = 100$ N
(C) $f_c = \sqrt{f^2 + N^2} = \sqrt{(100)^2 + (200)^2} = 100\sqrt{5}$ N

Q.15 (**B,C**)



T = mg
T = 100 mg sin 37° + 0.3 × 100 g cos 37°
[Put g = 9.8]
T = 588 + 235.2
mg = 823.2 ⇒ m = 82.33 = 83 kg

(b)



T + f = ma
T + 235.2 = 588
T = 588 – 235.2 = 352.8
m = 35.28 kg

Q.16 (**A,B**)

$f_{static_{max}} = 15 = \mu_s N$

$\mu_s = \frac{15}{N} = \frac{15}{mg} = \frac{15}{25} = 0.6$

Now let μ_k then
$15 - f_r = ma \Rightarrow 15 - \mu_k\, 25 = 2.5\,a$

$\mu_k = \frac{15 - 2.5a}{2.5}$ ...(1)

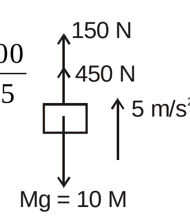
Now $x = ut + \frac{1}{2}at^2 \Rightarrow 10 = 0 + \frac{1}{2} \times a \times (5)^2$

$\Rightarrow a = \frac{10 \times 2}{5 \times 5} = \frac{4}{5} \Rightarrow a = \frac{4}{5}\,m/s^2$

$\therefore \mu_k = \frac{15 - 2.5 \times 4/5}{2.5} = 0.52$

Q.17 (B)
FBD of Block in ground frame :
applying N.L.
150 + 450 – 10 M = 5M

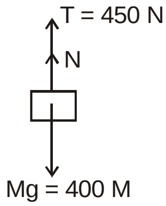
$\Rightarrow$ 15 M = 600 $\Rightarrow M = \frac{600}{15}$



⇒ M = **40 Kg Ans.**

Normal on block is the reading of weighing machine i.e. 150 N.

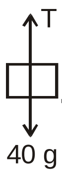
Q.18 (D)
If lift is stopped & equilibrium is reached then



$450 + N = 400$

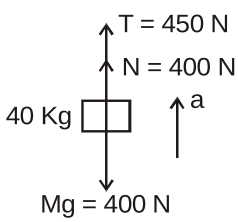
$\Rightarrow N = -50$

So block will lose the contact with weighing machine thus reading of weighing machine will be zero.



T = 40 g So reading of spring balance will be 40 Kg.

Q.19 (**A**)



$$\mathbf{a} = \frac{950 - 400}{40}$$

$$\Rightarrow a = \frac{450}{40} = \frac{45}{4} \text{ m/s}^2 \ \textbf{Ans.}$$

Q.20 (C)

$$a_p = \frac{10\,t}{10} = t$$

$$\therefore \frac{dv}{dt} = t$$

$$\Rightarrow \int_0^v dv = \int_0^t t\,dt$$

$$\Rightarrow v = \frac{t^2}{2}$$

Putting v = 2 we have t = 2 sec.

$$\text{Now } \frac{dx}{dt} = \frac{t^2}{2} \therefore x_p = \left[\frac{t^3}{6}\right]_0^2 = \frac{4}{3}$$

$x_B = 2 \times 2 = 4$ m

Hence relative displacement $= 4 - \frac{4}{3} = \frac{8}{3}$ m

Q.21 (B)

From above

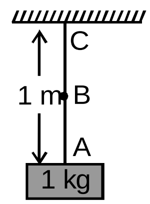
$$2t = \frac{t^3}{6} \Rightarrow t^2 = 12 \quad \Rightarrow t = 2\sqrt{3} \text{ sec.}$$

Q.22 (**C**)

$a = t = 4 \therefore$ after 4 seconds $V_B = 2$ m/s

$$V_p = \frac{4^2}{2} = 8 \text{ m/s} \therefore V_{rel} = 8 - 2 = 6 \text{ m/s.}$$

Q.23 (A)



Tension at A

$T_A = mg = 10$ N

Q.24 (B)

Tension at B

(mass of length AB = $\frac{1}{2}$ Kg)

$T_B = 1\,g + 0.5\,g = 15$ N

Q.25 (C)

(mass of length AC = 1 Kg)

Force exertd by support = T_C

= 1 g + 1 g = 20 N

Q.26 (A)

S_1 is accelerating frame so psuedo force act opposite to frame acceleration

F_{Pseudo} = mass of analyzing body × acceleration of frame

$$= 2(-5\hat{i} - 10\hat{j}) = -10\hat{i} - 20\hat{j}$$

Q.27 (B)

S_2 is inertial frame

F = ma

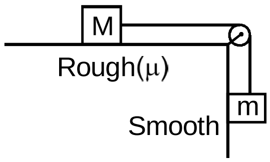
So $F = 10\hat{i} + 20\hat{j}$

Q.28 (A)

With respect to S_1 frame

Net force = zero.

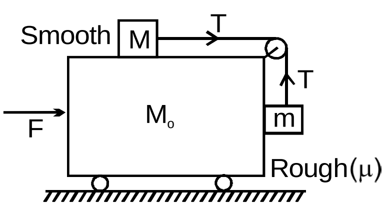
Q.29 (C)



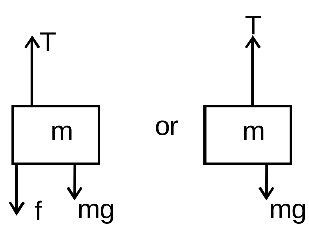
mg > μ M g
m > μM

Q.30 (A,C)
(A) m < μM
system is at rest T = mg
mg – T = ma
⇒ T = mg – ma
& T – μ Mg = Ma {m > μM}
⇒ T = Ma + μ M g
on analysing μ Mg < T < mg

Q.31 (A, D)

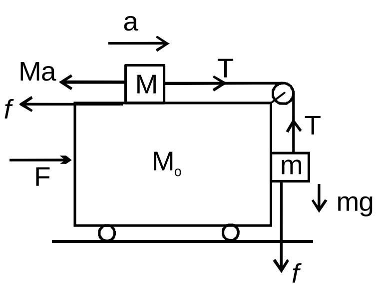


(A) When F = 0
No frictin b/w m & M_0 so system move.
(B) When F is applied then friction develope a range for which M and m are stationary w.r.t M_0, such that



(C) Limiting friction between M_0 & m is μ ma
∴ Dependent on a
(D) When Pseudo acts on M is equal to T
then f = 0

Q.32 (BC)



Use Pseudo concept
T = Ma ...(1)
T = f + mg
⇒ T = μ ma + ma ...(2)
On using (1) & (2)
Ma = μ ma + mg

Q.33 (A) p, r (B) p, r (C) q, s (D) q, r
(A) Let the horizontal component of velocity be u_x. Then between the two instants (time interval T) the projectile is at same height, the net displacement (u_xT) is horizontal

$$\therefore \text{ average velocity} = \frac{u_xT}{T} = u_x \Rightarrow \text{(A)} \quad \text{p, r}$$

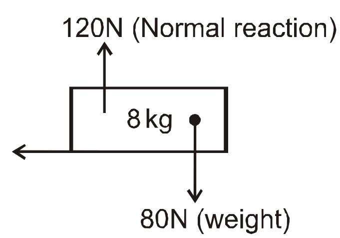
(B) Let $\hat{i}$ and $\hat{j}$ be unit vectors in direction of east and north respectively.

$$\therefore \ \vec{V}_{DC} = 20\hat{j}, \ \vec{V}_{BC} = 20\hat{i} \text{ and } \vec{V}_{BA} = -20\hat{j}$$

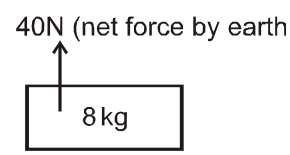
$$\therefore \ -\vec{V}_{AD} = \vec{V}_{DC} + \vec{V}_{CB} + \vec{V}_{BA} = 20\hat{j} - 20\hat{i} - 20\hat{j} = -20\hat{i}$$

$$\therefore \ \vec{V}_{AD} = 20\hat{i} \text{ Hence } \vec{V}_{AD} = \vec{V}_{BC} \Rightarrow \text{(B)} \quad \text{p, r}$$

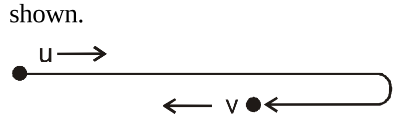
(C) Net force exerted by earth on block of mass 8 kg is shown in FBD and normal reaction exerted by 8 kg block on earth is 120 N downwards.



⇒



Hence both forces in the statement are different in magnitude and opposite in direction. ⇒ (C) q,s
(D) For magnitude of displacement to be less than distance, the particle should turn back. Since the magnitude of final velocity (v) is less than magnitude of initial velocity (u), the nature of motion is as shown.



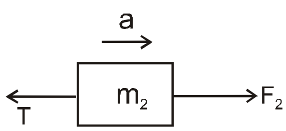
∴ Average velocity is in direction of initial velocity and magnitude of average velocity = $\frac{u - v}{2}$ is less than u because v < u. $\Rightarrow$ (C) q, r

Q.34 (A) q (B) r (C) q (D) r

Let a be acceleration of two block system towards right

$$\therefore a = \frac{F_2 - F_1}{m_1 + m_2}$$

The F.B.D. of m_2 is



$\therefore F_2 - T = m_2 a$

$$\text{Solving } T = \frac{m_1 m_2}{m_1 + m_2}\left(\frac{F_2}{m_2} + \frac{F_1}{m_1}\right)$$

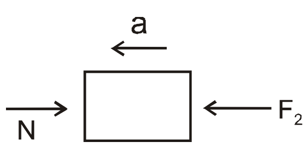
(B) Replace F_1 by $-F_1$ is result of A

$$\therefore T = \frac{m_1 m_2}{m_1 + m_2}\left(\frac{F_2}{m_2} - \frac{F_1}{m_1}\right)$$

(C) Let a be acceleration of two block system towards left

$$\therefore a = \frac{F_2 - F_1}{m_1 + m_2}$$

The FBD of m_2 is



$\therefore F_2 - N_2 = m_2 a$

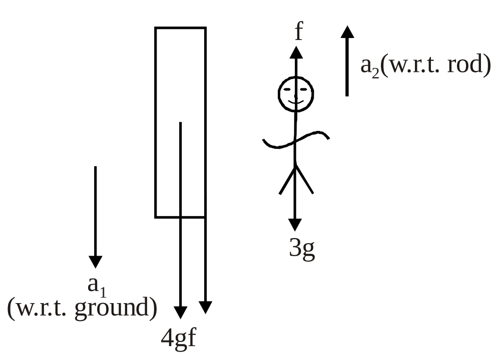
$$\text{Solving } N = \frac{m_1 m_2}{m_1 + m_2}\left(\frac{F_1}{m_1} + \frac{F_2}{m_2}\right)$$

(D) Replace F_1 by $-F_1$ in result of C

$$N = \frac{m_1 m_2}{m_1 + m_2}\left(\frac{F_2}{m_2} - \frac{F_1}{m_1}\right)$$

NUMERICAL VALUE BASED

Q.1 **[7]**



$$\vec{A}_{cat/g} = -a_1\hat{j} + a_2\hat{j} = \vec{0}$$

$a_1 = a_2 = a$ ...(i)

$4g + f = 4a$ ..(ii)

$3a + f - 3g = 3a$ ..(iii)

on solving above equation

$$a = 7\left(\frac{g}{4}\right)$$

n = 7

Q.2 [0020]

For the dog,

$N + T\sin\theta - Mg = 0$ (vertical)

$F - T\cos\theta = 0$ (horizontal)

For maximum extension, $f = \mu N$

For spring, $T\cos\theta - kx = 0$ (horizontal)

We have four unknowns (N, T, F, x) and four equations.

Solve for T:

$T\cos\theta = kx$

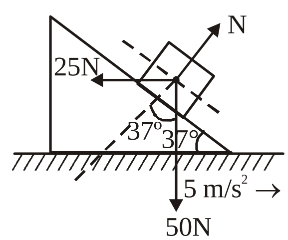
$\Rightarrow T = kx/\cos\theta$

Substitute for F and solve for N

$\mu N - T\cos\theta = 0$

$\Rightarrow N = T\cos\theta/\mu = kx/\mu$

Q.3 [55]



Considering motion of the block w.r..t the inclined plane

Pseudo force on block $F_P = 25$ N

Applying Newton's law on the block in direction perpendicular to the inclined surface,

N = 25 sin 37° + 50 cos 37°

N = 15 + 40

N = 55 Newton

By Newton's Third law,

this force exerted by inclined plane on the block is equal to the force exerted by the block on the inclined plane .

Q.4 [8]

For equilibrium,

$10 = 8 + T$ …(i)

$T + f_2 = 20$ …(ii)

$\Rightarrow f_2 = 18N$

Q.5 [10 sec]

$a_x = \dfrac{6}{1+5} = 1\ ms^{-2}$

$N = 1 \times 1 = 1\ N$

$f_r = \mu N = \dfrac{1}{2}$

$a = \dfrac{fr}{m} = \dfrac{1}{2}$

$0 = v_0 - \dfrac{1}{2} \times t$

$t = 10$ sec.

Q.6 [4]

$a = 4t$

at t = 1 sec, slipping occure

$ma = \mu_s mg$

$4t = \mu_s g$

$\mu_s = 0.4$

$t = 1 \rightarrow t = 3$ slipping occur

b/w t = 1 & t = 2sec

$(4t - \mu_k g) = \dfrac{dv}{dt}$

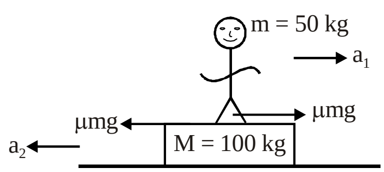
$v = \left[2t^2 - \mu_k gt\right]_1^2$

$v = 2 \times 3 - \mu_k g$

$\mu_k = 0.3\ ,\ \dfrac{3\mu_s}{\mu_k} = 4$

Q.7 [10]

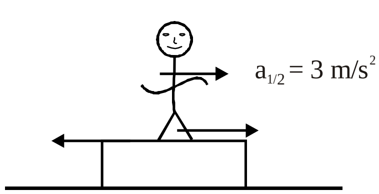
a_1 and a_2 w.r.t. ground



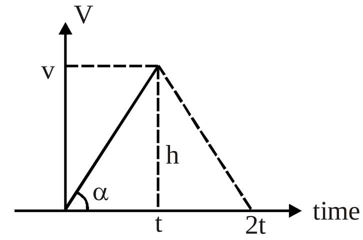
$a_1 = \dfrac{\mu mg}{m} = 2 m/s^2$

$a_2 = \dfrac{\mu mg}{M} = 1 m/s^2$

Plank as frame of reference



Plank comes to rest



$\tan\alpha = 3 = \dfrac{h}{t}$

$h = 3t$

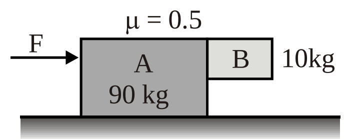
for minimum time, man will acceleration and retard with $a_{1/2}$ for equal time interval t.

Area of v vs t graph = displacment

$\dfrac{1}{2}(2t)(3t) = 75$ $\qquad t = 5$

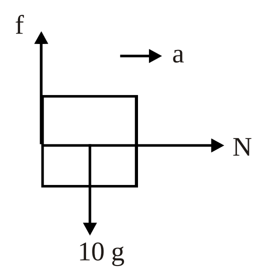
minimum time = 2t = 10 sec

Q.8 [2000 N]



acceleration of the blocks, $a = \dfrac{F}{100}$

FBD of B,



Applying Newton's II Law in horizontal for block B,

$N = 10 \times \dfrac{F}{100}$

For limiting condition $f = \mu N$

$f = 10\ g$

$\therefore \quad \mu N \geq 10g$

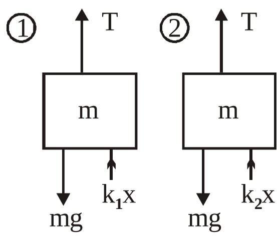
$0.5 \times \dfrac{F}{10} \geq 10\ g$

$\therefore \quad F \geq 2000$ N.

KVPY

PREVIOUS YEAR'S

Q.1 (A)



$T = k_2 x - mg = ma_1$

$k_2x + mg - T = ma_2$

By constraint relation

$a_1 = a_2$

Q.2 (B)

F = Mg = T

When spring is cut $a_M = 0$

For M 

$\Uparrow a_m = \left(\frac{M-m}{m}\right)g$

Q.3 (B)

Let $f = Kt^n$

At t = 2 F = 2 = $K2^n$

t = 4 $\quad$ F = 16 = K^4

$$\Rightarrow \frac{16}{2} = \left(\frac{4}{2}\right)^n = 2^n \Rightarrow n = 3$$

$$F = K2^3 = \Rightarrow K = \frac{1}{4}$$

$$F = \frac{t^3}{4}$$

$$dp = \int_0^t Fdt = \frac{t^4}{16} = m(V_f - V_t)$$

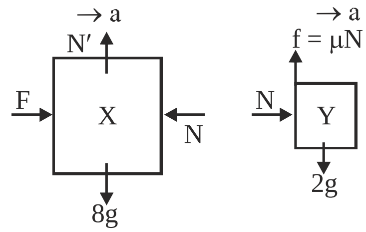
$$t = 3 \text{ sec} \quad \frac{81}{16} = 2m$$

$$t = 4 \quad \frac{256}{16} = mV_f$$

$$\frac{V_t}{2} = \frac{256}{81} \Rightarrow V_f = 6.32$$

Q.4 (A)

According to free body diagram



F – N = 8a .....(1)

N = 2 a ....(2)

f = 2g = 20 ....(3)

$\Rightarrow \mu N = 20$

$\Rightarrow N = \frac{20}{\mu} = \frac{20}{0.5} = 40$ .....(4)

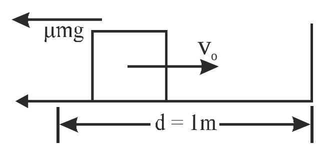
$\Rightarrow N = 2a$

$\Rightarrow a = \frac{N}{2} = \frac{40}{2} = 20 \text{ m/s}^2$ .....(5)

F = N + 8a = 10 a [from equation (1)]

F = 10 × 20 = 200 newton

Q.5 (A)



$a = \mu g$

Velocity after time t

$v = v_0 - \mu gt$

$v \geq 0 \Rightarrow v_0 \geq \mu gt$ ....(i)

Displacement in time 't'

$$\Delta X = v_0 t - \frac{1}{2}\mu g t^2$$

$$1 = 2v_0 - \frac{1}{2}\mu g(2)^2$$

$$v_0 = \left(\frac{1}{2} + \mu g\right) \quad \text{....(ii)}$$

$\frac{1}{2} + \mu g > 2\mu t$ $\quad$ (t = 2 sec)

$$\frac{1}{2} + \mu g > 2\mu g$$

$$\mu g < \frac{1}{2}$$

$$\mu < \frac{1}{2g}$$

$$\mu < 0.05$$

JEE-MAIN
PREVIOUS YEAR'S

Q.1 (1)

$a_x = \frac{20}{2} = 10 \text{ m/s}^2$

$x = 0 + \frac{1}{2} \times 10 \times 10^2 = 500 \text{ m}$

Q.2 [492]

mg – V T = ma

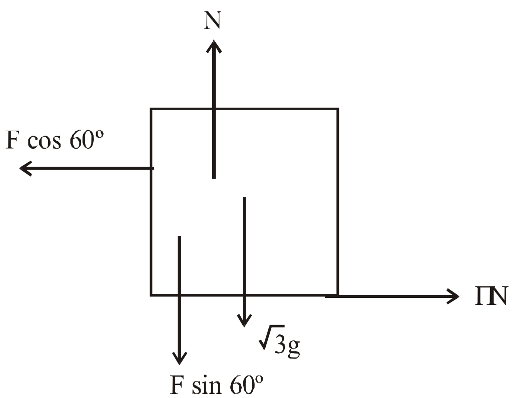
T = m(g – a)

= 60 [10 – 1.8]

= 60 × 8.2

= 492 N

Q.3 [3]



$F\cos 60^\circ = \mu(3g + F\sin 60^\circ)$

$\therefore \quad \frac{F}{2} = \mu\left(\sqrt{3}g + \frac{\sqrt{3}F}{2}\right)$

$\therefore \quad \frac{F}{2} = \frac{\sqrt{3}g}{3\sqrt{3}} + \frac{F}{6}$

$\therefore \quad \frac{F}{3} = \frac{\sqrt{3}g}{3\sqrt{3}}$

F = g = 10

3x = 10

$x = \frac{10}{3}$

= 3.33

Q.4 [25 cm]

$\mu \geq \tan\theta = \frac{dy}{dx} = \frac{dx}{4} = \frac{x}{2}$

$0.5 \geq \frac{x}{2}$

$x \leq 1$

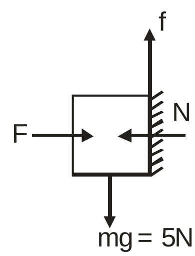
$\sqrt{4y} \leq 1$

$2\sqrt{y} \leq 1$

$y \leq \frac{1}{4}$

Q.5 [25]

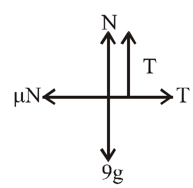
F = N, f = 0.2 × N



0.2 N ≤ 5

N ≤ 25

Q.6 [30]



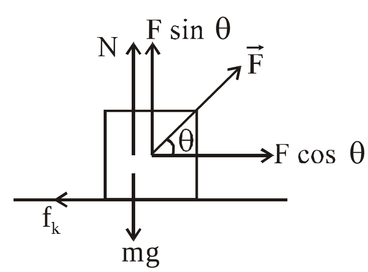
N + T = 90

T = μ N = 0.5 (90–T)

1.5 T = 45

T = 30

Q.7 (2)



N = mg – f sin θ

$F\cos\theta - \mu_K N = ma$

$F\cos\theta - \mu_K(mg - F\sin\theta) = ma$

$a = \frac{F}{m}\cos\theta - \mu_k\left(g - \frac{F}{m}\sin\theta\right)$

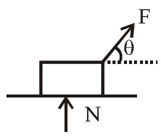
Q.8 [21]

$a_{max} = \mu g = \frac{3}{7} \times 9.8$

$F = (M + m)\, a_{max} = 5\, a_{max}$

$= 21$ Newton

Q.9 [5]



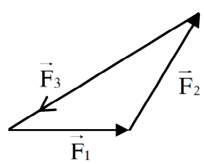
$F \cos \theta = \mu N$

$F \cos \theta + N = mg$

$\Rightarrow F = \frac{\mu mg}{\cos\theta + \mu \sin\theta}$

$F_{min} = \frac{\mu mg}{\sqrt{1+\mu^2}} = \frac{\frac{1}{\sqrt{3}} \times 10}{\frac{2}{\sqrt{3}}} = 5$

Q.10 (4)



Here $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$

$\vec{F}_1 + \vec{F}_2 = -\vec{F}_3$

Since $\vec{F}_{net} = 0$ (equilibrium)

Both statements correct

Q.11 (2)

Q.12 (1)

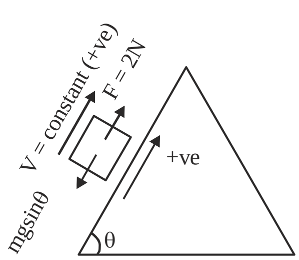
Q.13 (2)

Q.14 (3)

Q.15 (2)

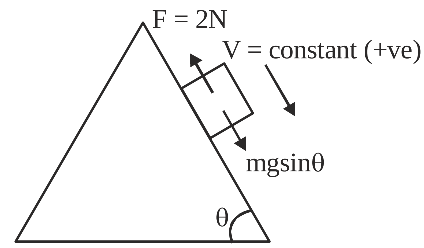
Q.16 (2)

During upward motion



F = 2N = (+ve) constant

During downward motion

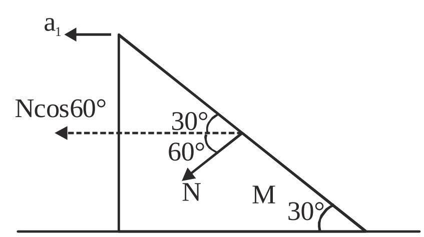


$\Rightarrow$ F = 2N = (–ve) constant

$\Rightarrow$ Best possible answer is option (2)

Q.17 (4)

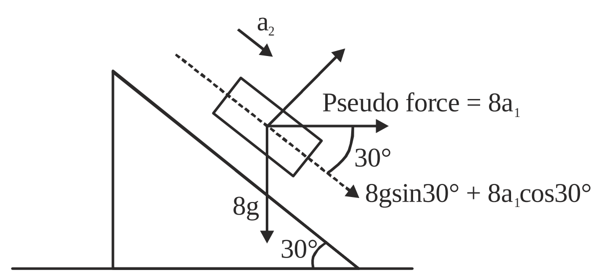
let acceleration of wedge is a_1 and acceleration of block w.r.t. wedge is a_2



$N \cos 60° = Ma_1 = 16a_1$

$\Rightarrow N = 32a_1$

F.B.D. of block w.r.t. wedge



$\perp$ to incline

$N = 8g \cos 30° – 8a_1 \sin 30° \Rightarrow 32a_1 = 4\sqrt{3}\, g – 4a_1$

$\Rightarrow a_1 = \frac{\sqrt{3}}{9} g$

Along incline

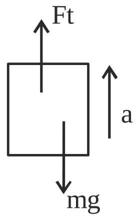
$8g\sin30° + 8a_1\cos30° = ma_2 = 8a_2$

$a^2 = g \times \frac{1}{2} + \frac{\sqrt{3}}{9} g . \frac{\sqrt{3}}{2} = \frac{2g}{3}$

Option (4)

Q.18 (4)

Q.19 (4)



$$F_{thrust} = \left(\frac{dm}{dt} \cdot V_{rel}\right)$$

$$\left(\frac{dm}{dt} V_{rel} - mg\right) = ma$$

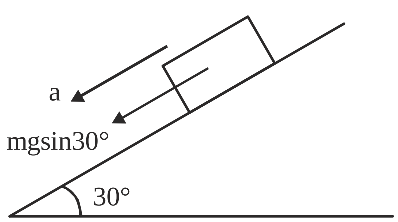
$$\Rightarrow \left(\frac{dm}{dt}\right) \times 500 - 10^3 \times 10 = 10^3 \times 20$$

$$\frac{dm}{dt} = (60 kg/s)$$

Option (4)

Q.20 [2]

Q.21 (3)

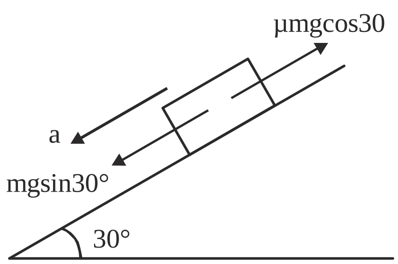


On smooth incline

a = g sin30°

by $S = ut + \frac{1}{2}at^2$

$$S = \frac{1}{2}\frac{g}{2}T^2 = \frac{g}{4}T^2 \text{........(i)}$$



On rough incline

a = g sin30° – µg cos 30°

by $S = ut + \frac{1}{2}at^2$

$$S = \frac{1}{4}g\left(1 - \sqrt{3}\mu\right)(\alpha T)^2 \text{ .....(ii)}$$

By (i) and (ii)

$$\frac{1}{4}gT^2 = \frac{1}{4}g\left(1 - \sqrt{3}\mu\right)\alpha^2 T^2$$

$$\Rightarrow 1 - \sqrt{3}g = \frac{1}{\alpha^2} \Rightarrow g = \left(\frac{\alpha^2 - 1}{\alpha^2}\right) \cdot \frac{1}{\sqrt{3}}$$

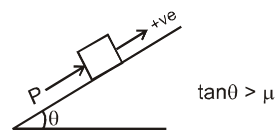
$\Rightarrow$ x = 3.00

Q.22 [3]

Q.23 [15]

JEE-ADVANCED PREVIOUS YEAR'S

Q.1 (A)

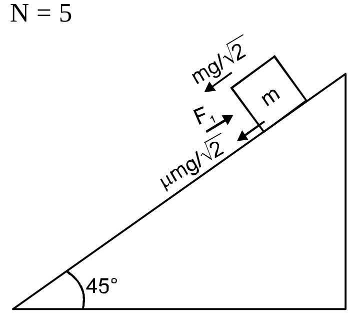


P_1 = mgsinθ – µmgcosθ

P_2 = mgsinθ + µmgcosθ

Initially block has tendency to slide down and as tanθ > µ, maximum friction µmgcosθ will act in positive direction. When magnitude P is increased from P_1 to P_2, friction reverse its direction from positive to negative and becomes maximum i.e.µmgcosθ in opposite direction.

Q.2 N = 5



$$F_1 = \frac{mg}{\sqrt{2}} + \frac{\mu mg}{\sqrt{2}}; \qquad F_2 = \frac{mg}{\sqrt{2}} - \frac{\mu mg}{\sqrt{2}}$$

$F_1 = 3F_2$

1 + µ = 3 – 3µ

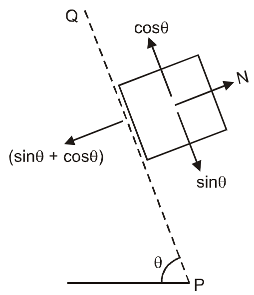
4µ = 2

$\mu = \frac{1}{2}$

N = 10µ

N = 5

Q.3 (A, C)



f = 0, If sinθ = cosθ ⇒ θ = 45°
f towards Q, sinθ > cosθ ⇒ θ > 45°
f towards P, sinθ < cosθ ⇒ θ < 45°

Q.4 (D)
Block will not slip if
$(m_1 + m_2)\, g \sin\theta \le \mu\, m_2\, g \cos\theta$

$$3 \sin\theta \le \left(\frac{3}{10}\right)(2) \cos\theta$$

$$\tan\theta \le \frac{1}{5}$$

$$\Rightarrow \quad \boxed{\theta \le 11.5^\circ}$$

(P) θ = 5° friction is static $f = (m_1 + m_2)g \sin\theta$
(Q) θ = 10° friction is static $f = (m_1 + m_2)g \sin\theta$
(R) θ = 15° friction is kinetic $f = \mu m_2 g \cos\theta$
(S) θ = 20° friction is kinetic $f = \mu m_2 g \cos\theta$

Q.5 (A, B, D)

$$\frac{dk}{dt} = \gamma t \ \textbf{as} \qquad k = \frac{1}{2}mv^2$$

$$\therefore \frac{dk}{dt} = mv\frac{dv}{dt} = \gamma t$$

$$\therefore m\int_0^v vdv = \gamma\int_0^t tdt$$

$$\frac{mv^2}{2} = \frac{\gamma t^2}{2}$$

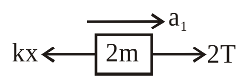
$$v = \sqrt{\frac{\gamma}{m}}\,t \qquad .....(i)$$

$$a = \frac{dv}{dt} = \sqrt{\frac{\gamma}{m}} = \text{constant}$$

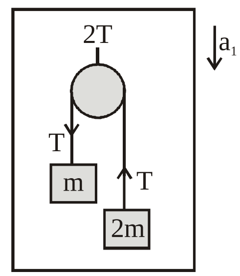
since F = ma

$$\therefore F = m\sqrt{\frac{\gamma}{m}} = \sqrt{\gamma m} = \text{constant}$$

Q.6 (C)



$2T - kx = 2ma_1$



$$T = \frac{2(2m)(m)}{3m}(g - a_1)$$

$$= \frac{4m}{3}(g - a_1)$$

$$\frac{8m}{3}(g - a_1) - kx = 2ma_1$$

$$\frac{8Mg}{3} - \frac{8ma_1}{3} - kx = 2ma_1$$

$$\frac{8Mg}{3} - kx = \frac{14ma_1}{3}$$

$$\frac{8Mg - 3kx}{14m} = a_1$$

$$a_1 = \frac{8mg - 3kx}{14m}$$

$$\frac{vdv}{dx} = \left(\frac{8Mg}{14m} - \frac{3kx}{14m}\right)$$

$$\int vdv = \frac{1}{14m}\int (8Mg - 3kx)dx$$

for max elongation

$$0 = \frac{1}{14m}\int_0^{x_0} (8mg - 3kx)dx$$

$$= \frac{1}{14m}\left(8Mgx_0 - \frac{3kx_0^2}{2}\right)$$

$$8Mgx_0 = \frac{3kx_0^2}{2}$$

$x_0 = \frac{16Mg}{3k}$

at $x = \frac{x_0}{2}$

$$\int_0^v vdv = \frac{1}{14m}\int_0^{x_0/2}(8Mg - 3kx)dx$$

$$\frac{v^2}{2} = \frac{1}{14m}\left(\frac{8Mgx_0}{2} - \frac{3kx_0^2}{2\times4}\right)$$

$$v^2 = \frac{1}{7m}\left(\frac{8Mg}{2}\times\frac{16Mg}{3x} - \frac{3x}{8}\times\frac{16M^2g^2}{3x\times3x}\right)$$

$$= \frac{1}{7m}\left(\frac{64M^2g^2}{3x} - \frac{2M^2g^2}{3x}\right)$$

$$v^2 = \frac{62Mg^2}{21k}$$

For acc. $2a_1 = a_2 + a_3$ therefore

$a_2 - a_1 = a_1 - a_3$

$$a_1 = \frac{8Mg - 3kx_0/4}{14m}$$

$$= \frac{8g}{14} - \frac{3kx_0}{14m\times4}$$

$$= \frac{8g}{14} - \frac{3x}{14m\times4}\times\frac{16Mg}{3x}$$

$$= \frac{8g}{14} - \frac{4g}{14}$$

$$= \frac{4g}{14} = \frac{2g}{7}$$

OR

$$\frac{8mg}{3} - \frac{8m}{3}a_1 - kx = 2ma_1$$

$$\frac{14m}{3}a_1 = -k\left[x - \frac{8mg}{3k}\right]$$

$$a_1 = -\frac{3k}{14m}\left[x - \frac{8mg}{3k}\right] \quad .....(i)$$

that means, block 2m (connected with the spring) will perform SHM about $x_1 = \frac{8mg}{3k}$ therefore.

maximum elongation in the spring $x_0 = 2x_1 = \frac{16mg}{3k}$

on comparing equation (1) with

$a = -\omega^2(x - x_0)$

$$\omega = \sqrt{\frac{3k}{14m}}$$

at $\left(\frac{x_0}{2}\right)$, block will be passing through its mean position therefore at mean position

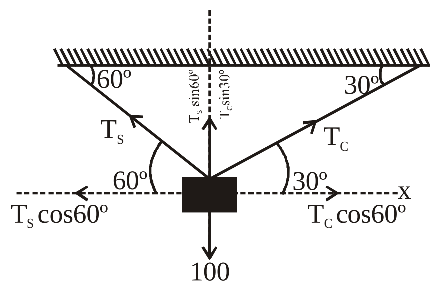
$$v_0 = A\omega = \frac{8mg}{3k}\sqrt{\frac{3k}{14m}}$$

At. $\frac{x_0}{4} \Rightarrow x = \frac{A}{2}$

$$\therefore a_{cc} = -\frac{A}{2}\omega^2$$

$$= -\frac{4mg}{3k}.\frac{3h}{14m} = \frac{2g}{7}$$

Q.7 (2.00)



Let T_S = tension in steel wire

T_C = Tension in copper wire

in x direction

$T_C \cos30° = T_S \cos60°$

$$T_C \times \frac{\sqrt{3}}{2} = T_S \times \frac{1}{2}$$

$$\sqrt{3}\,T_C = T_S \quad ......(i)$$

in y direction

$T_C \sin30° + T_S \sin60° = 100$

$$\frac{T_C}{2} + \frac{T_S\sqrt{3}}{2} = 100 \quad .....(ii)$$

Solving equation (i) & (ii)

$T_C = 50$ N

$T_S = 50\sqrt{3}$ N

We know

$$\Delta L = \frac{FL}{AY}$$

$$= \frac{\Delta L_C}{\Delta L_S} = \frac{T_C L_C}{A_C Y_C}\times\frac{A_S Y_S}{T_S L_S}$$

On solving above equation

$$\frac{\Delta L_C}{\Delta L_S} = 2$$

Ans. 2.00